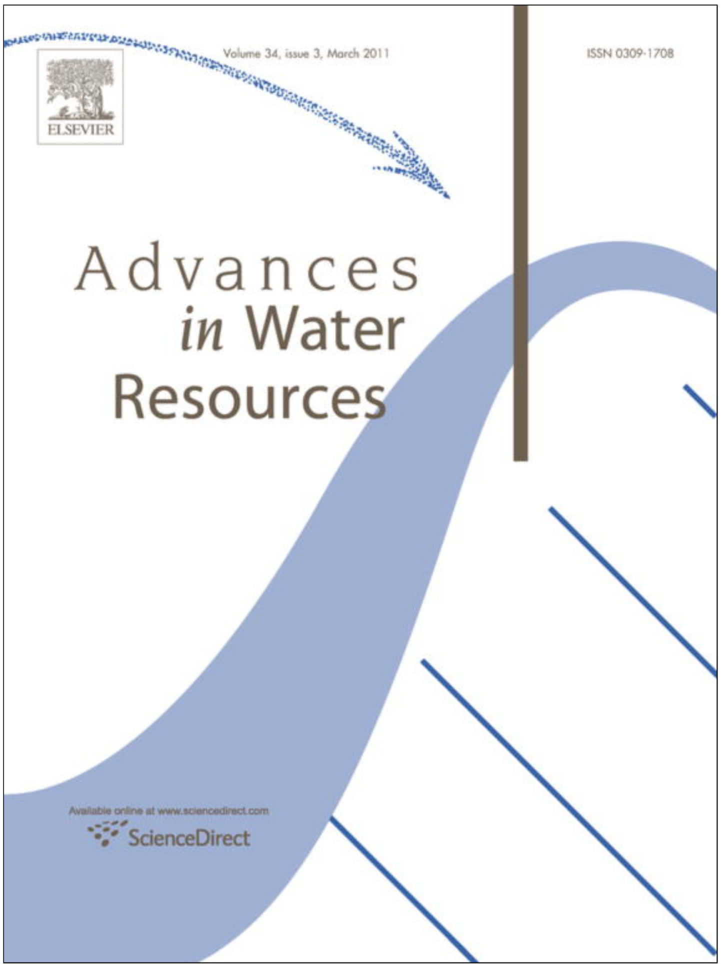
Volume 34, issue 3, March 2011
ISSN 0309-1708
ELSEVIER
Advances in Water Resources
Available online at www.sciencedirect.com
ScienceDirect

Advances in Water Resources 34 (2011) 407–416

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

# Parameter estimation in ensemble based snow data assimilation: A synthetic study

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ARTICLE INFO

*Article history:*
Received 8 November 2009
Received in revised form 24 September 2010
Accepted 2 December 2010
Available online 10 December 2010

*Keywords:*
Parameter estimation
Snow hydrology
Data assimilation

ABSTRACT

Estimating erroneous parameters in ensemble based snow data assimilation system has been given little attention in the literature. Little is known about the related methods' effectiveness, performance, and sensitivity to other error sources such as model structural error. This research tackles these questions by running synthetic one-dimensional snow data assimilation with the ensemble Kalman filter (EnKF), in which both state and parameter are simultaneously updated. The first part of the paper investigates the effectiveness of this parameter estimation approach in a perfect-model-structure scenario, and the second part focuses on its dependence on model structure error. The results from first part research demonstrate the advantages of this parameter estimation approach in reducing the systematic error of snow water equivalent (SWE) estimates, and retrieving the correct parameter value. The second part results indicate that, at least in our experiment, there is an evident dependence of parameter search convergence on model structural error. In the imperfect-model-structure run, the parameter search diverges, although it can simulate the state variable well. This result suggest that, good data assimilation performance in estimating state variables is not a sufficient indicator of reliable parameter retrieval in the presence of model structural error. The generality of this conclusion needs to be tested by data assimilation experiments with more complex structural error configurations.



## 1. Introduction

As an important land feature, snow affects the land surface energy balance via its unique thermodynamic properties (e.g., high albedo, low thermal conductivity) that vary spatially and temporally. Snow is also an important freshwater resource, as runoff from snow melting is indispensable for many drought prone regions in spring and summer. Accurately characterizing snow conditions is critical for hydrological forecast, diagnosing hydroclimatologic trends and subsystem interactions (e.g., land snow cover–atmosphere interaction), and monitoring floods and droughts, among other applications. Although this need is pressing, the in situ observation of snow has been limited, in part because the cold environment is often prohibitive for extensive ground survey efforts.

In recent years data assimilation methods have provided a new opportunity for snow estimation. In particular, the ensemble Kalman filter (EnKF) methods have been used to estimate snow water equivalent (SWE) and other cold region variables (e.g., [7,13,12]). Various work has focused on different spatial scales, while using the same error-covariance based algorithm to sequentially adjust model simulations. Theoretically, the EnKF method requires that model and observation function (the function relating observations and model simulations) are free of systematic error to accurately propagate ensemble and avoid bias in the update equation. However, land surface model (LSM) and observational functions applied in the above studies may have system errors that originate from a broad range of sources including, e.g., incomplete representation of snow properties and associated dynamical processes, uncertainties in model parameters, and scaling or physical discrepancy [4] between observation and model estimates, etc. Each of the related sources is linked to a different stage of model development or remote sensing data processing. The mixture of these errors in the highly non-linear LSM simulation makes their individual effect hard to isolate and interpret [9]. Affected by these uncertainties, the snow data assimilation system is vulnerable to systematic error and unrealistic updates. In particular, parametric errors are common in LSMs and observational functions, yielding negative effects on the EnKF algorithm. One significant consequence could be a mismatch between model predication and observation, since the (artificial) model systematic errors are neither represented in observations nor dealt with in the traditional ensemble perturbation scheme in the EnKF.

To tackle this issue, recent studies have examined the impacts of parameter uncertainty in multi-scale snow simulation that includes electromagnetic signatures [3]. However, few studies have focused on a method to correct parameter uncertainty in ensemble snow data assimilation systems. In most of the EnKF experiments, those critical parameters characterizing snowpack dynamics were simply treated as perfectly known or by inflating with prescribed noise. Viewed in a boarder context, the issue of parameter error

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0309-1708/$ - see front matter 
doi:10.1016/j.advwatres.2010.12.002

in ensemble data assimilation has been investigated in some hydrological and meteorological studies (e.g., [1,2,16,20,23]). A common strategy adopted in these studies is to combine parameters and states in an "augmented vector" and reduce the parameter-state optimization to a state-variable filtering problem. Following the same general method, this study concentrates on reducing parameter uncertainties in a snow model forced with prescribed meteorological data.

In this research we also investigate how state-space based parameter estimation is dependent on the model physical structures (the physical structural error in the system). By definition, the model structure refers to model physical and dynamical formulation and parameterization. Currently it is unclear weather the parameter estimation with problematic model structure can achieve comparable performance (compared to results from those using perfect-structure model) in updating snowpack state variables and parameters. The insights gained from addressing this question benefit the snow data assimilation in a broad sense. Currently there is a wide range of physical structures (including parameterization) in different snow models [14], reflecting the distinct (or incompatible) perspectives taken to understand the same real world process. These different perspectives are usually results of limited predicative capability for a given model structure, and the availability and quality of measurements. It is difficult to fully rank the physical appropriateness of these model structures, and each model structure (describing physically equivalent process) may only capture part of the truth. Understanding the impact of their differences on the parameter estimation is important to evaluating the sensitivity of snow data assimilation performance to model structure error. Further, according to Clark [9], for streamflow simulation at given climate regime, the selection of the model structure could be just as important as selection of the model parameters. Our research concerns whether parameter estimation can fully compensate for the model structural error in the EnKF snow estimation system. If not, what is the signature of model structural error? Could it influence the identification of the parameters for individual model structure given that true (or most appropriate) structures and parameters are often unknown (or partially known) for the tested region?

This paper is structured as two parts. First the performance of simultaneous state and parameter estimation in a synthetic EnKF snow data assimilation system is comprehensively evaluated. In this part the model physical structure is assumed to be perfectly known. Built on the similar data assimilation infrastructure, a relatively simple model structure error is introduced in the second part and the performance of parameter estimation is compared to that of the perfect-model-structure run. The next section introduces the LSM, the EnKF method, and the parameter estimation scheme. Two different snow data assimilation experiments are described in Sections 3 and 4. Discussions and results are given in Section 5 with concluding remarks in Section 6.

## 2. Methodology

The Community Land Model (CLM 2.0, [5]) is used to propagate land state variables. CLM numerically simulates energy, momentum and water exchanges between the land surface and the overlying atmosphere. It employs 10 soil layers to resolve soil moisture and temperature dynamics and uses plant functional types (PFTs) to represent sub-grid vegetation heterogeneity. Its snow model simulates a snowpack with multiple layers (1–5 layers) depending on its thickness, and accounts for processes such as liquid water retention, diurnal cycling of thawing–freezing, snowpack densification, snow melting, and surface frost and sublimation. Besides these features, the CLM used in this research includes a new snow cover fraction (SCF) parameterization which dynamically adjust the relationship between grid averaged SWE and SCF [19]. Estimating a related parameter in this parameterization is one of the central topics in our experiments. More discussion about this parameter and the associated equation is given in Section 3. The general performance of CLM2.0 in cold region simulations is given by Bonan et al. [5] and Niu et al. [18,19].

The EnKF was first introduced by Evensen [15] as a Monte Carlo approach to accomplish the Kalman filter updating scheme in numerical modeling systems. It uses ensemble approach to represent model errors (e.g., forcing error). It propagates estimations of errors in state and fluxes variables with model physics in temporal and spatial spaces, and updates these estimations in state space with conditional distribution approach (Bayesian formulation). In this paper the EnKF is implemented as follows: (a) each sample (i.e., ensemble member) of model state variables is propagated at every time step using prognostic equations; these simulations are driven by perturbed meteorological forcing data (the method of sampling forcing is introduced in Section 3); (b) each sample of the LSM forecast variables is updated (e.g., SWE in this study) using Eq. (1):

$$x_{i,t}^a = x_{i,t}^b + K_t(y_t - H(x_{i,t}^b + v_i)) \tag{1}$$

where $x_{i,t}^a$ denotes the filter updated states (e.g., SWE), $x_{i,t}^b$ the model simulated states, $i$ the ensemble index, $y_t$ the observation, and $H$ the observational operator. $v_t^i$ is randomly drawn from a Gaussian distribution (with zero mean and the variance equal to $R_t$ as described below) to ensure an adequate spread of the analysis ensemble members [6]. Additional discussion of the EnKF scheme in CLM is given by Su et al. [22]. Modifications to include parameter estimation are incorporated in the same mathematical framework. Eq. (2) gives a brief statement of the parameters-augmented EnKF analysis

$$([x_{i,t}, \delta_i]^T)^a = ([x_t, \delta_i]^T)^b + K_t(y_t - H([x_{i,t}, \delta_i]^T)^b + v_i) \tag{2}$$

where the symbols have the same meaning as in Eq. (1) and $\delta$ represents parameter vectors.

To deal with the problems of variance reduction in parameter ensemble space and filter divergence, we adopt the "conditional covariance inflation" method proposed by Aksoy et al. [1], which prescribes a threshold for the parameter ensemble variance, without augmenting the ensemble spread at each system propagation step. Eq. (3) gives the formulation of this inflation method, i.e., the evolution equation of the parameter ensemble $\delta$

$$\begin{aligned} &\delta_{t+1}^b = \delta_t^a \quad \text{if } Var(\delta_t^a) \geqslant Q_0 \\ &\delta_{t+1}^b = \delta_t^a + \varepsilon \quad \text{if } Var(\delta_t^a) \leqslant Q_0,\ \varepsilon \in N(0, Q_0 - Var(\delta_t^a)) \end{aligned} \tag{3}$$

In which $N(0, Q_0 - Var(\delta_t^a))$ represents the normal distribution with zero mean and variance of $Q_0 - Var(\delta_t^a)$. $Q_0$ is the prescribed variance threshold. This inflation approach has been shown successful in a highly non-linear and flow-dependent meteorological data assimilation system [1], where the proper constraints on parameter ensemble variance are shown to be important for the accurate EnKF update. Because there is no prior knowledge about the evolution of parameters, $Q_0$ largely controls the parameter ensemble trajectory. The selection of $Q_0$ and its implications are discussed in following sections.

## 3. The performance of parameter estimation with perfect-model-structure

The first group of simulations is a synthetic experiment in which a single grid simulation for six months (November–April) is created with CLM driven by ensemble meteorological forcing.

In the experiment, the true state of related variables is known. Table 1 provides a list of these simulations. The model time step is 30 min, which is small enough to characterize major snowpack dynamic processes. For simplicity, only one vegetation tile type (grass) is used in the grid. Therefore the parameters involved in the forest–snowpack thermodynamic interactions are neglected. In addition, the subgrid snow cover heterogeneity is included. Since the patch of snow cover can drive similar process across a diverse range of scales in influencing snowpack radiative energy balance, the related experiments in this one-dimensional synthetic study can have some degree of spatial representativeness. Ensemble forcing is produced by perturbing nominal precipitation and air temperature with 50% and 3 °C (rms) Gaussian error, respectively. The ensemble runs contain 59 members and an additional run is conducted to represent synthetic truth. The forcing data to construct the synthetic true values are arbitrarily taken from the same sampling set that generates the ensemble run. This makes the forcing error parameters assumed in the EnKF capture the statistics of true uncertainty. The resultant synthetic true values provide a benchmark for quantitatively understanding parameter estimation effects. Synthetic SWE observations are produced every two days by adding Gaussian perturbations (20 mm rms) to the true state.

We consider estimation of two parameters that are typically difficult to determine from measurements. These are: (1) the exponent $\alpha$ in the snow cover fraction parameterization described by Eq. (4) [19]; and (2) the liquid water holding capacity $\theta$ (the maximum volume, in percentage, of liquid water in snowpack, Eq. (6)). The parameter $\alpha$ governs the relationship between the snow depth (or SWE) and the snow cover fraction (SCF), strongly influencing grid averaged albedo and snowpack energy processes (Eq. (5)). Niu and Yang [19] have demonstrated the significant sensitivity of CLM performance in simulating cold region variables to this parameterization (4) and the parameter $\alpha$, implying their potential importance to the snow data assimilation

$$SCF = \tanh\left(\frac{h_{sno}}{2.5z_0(\rho_{sno}/\rho_{new})^{\alpha}}\right) \quad (4)$$

$$A_{total} = A_{snow} * SCF + A_{grass} * (1 - SCF) \quad (5)$$

$$\theta_{drainage} = \theta_{liquid-snowpack} - \theta, \quad \text{if } \theta_{liquid-snowpack} > \theta \quad (6)$$

Here $h_{sno}$ and $z_0$ are the snow depth and the ground surface roughness length, respectively; $\rho_{new}$ is a prescribed fresh snow density; and $\rho_{sno}$ is the model calculated snow density [19]. $A_{total}$ is the grid averaged albedo to calculate the ground energy balance, $A_{snow}$ and $A_{grass}$ are the snow and grass albedo, respectively (with $A_{snow}$ much larger than $A_{grass}$). With other conditions unchanged, increasing $\alpha$ generally leads to a decrease of SWE, and vice versa. The parameter $\theta$ sets the threshold for the variability of liquid water content in the snowpack, thus controlling its thermodynamic properties, stratigraphic characteristics (e.g., snow density), and melting process. With other conditions unchanged, increasing $\theta$ generally leads to an increase of SWE, and vice versa. In previous research, this parameter is often given an empirical number without quantitative method to derive application specific value. The main reason could be that there is few physical law, if any, to estimate $\theta$. While, it can exert significant effects on the dynamical processes. In Section 3, it would be shown how an erroneous $\theta$ can influence the simulation of snowpack.

A further reason to select these parameters is that we assume they are representative for the parametric uncertainty in this nonlinear model structure, and the demonstrated system behaviors could be typical in the scope defined by our central objectives in Section 1. Considering that our purpose here is to preliminarily assess the overall performance of the parameter estimation approach and not to enumerate parameters in different components of the model and comprehensively characterize their joint estimation, increasing the number of analyzed parameters may be less important than the need of keeping the system tractable by limiting the size of parameter vector. Accordingly the interaction among multiple parameters and their differing ensemble trajectories are not focused in this research (while the relationship between above two parameters and their different impacts to data assimilation are interpreted), although we recognize the potential importance of analyzing larger parameter vector in obtaining more insights of the system.

There are six simulations designed in this section (see Table 1 for description of these simulations). In SYN_TRUE the true values of $\alpha$ and $\theta$ are 1.6 and 0.01, respectively. For the imperfect model simulations (SYN_CR, DA_PAR_CR, DA_PAR_$\alpha$, DA_PAR_$\theta$) $\alpha$ and $\theta$ are set as 0.6 and 0.08, respectively, representing parameter errors. According to Aksoy et al. [1], the initial standard deviation for parameter ensemble is set to the difference between the initial mean parameter value and the true parameter value (1.0 and 0.07 for $\alpha$ and $\theta$, respectively), while the prescribed standard deviation (square root of $Q_0$ in Eq. (2)) is set to 1/4 of the initial standard deviation. Other $Q_0$ values have been tested and the results of both variables and parameters are very similar (refer to figure in Section 4 for more details about this sensitivity). So we present the results with the above $Q_0$ which is representative.

Fig. 1 compares ensemble mean results of SWE from SYN_CR, DA_PAR_CR and SYN_TRUE. Table 2 gives Nush–Sutcliffe efficiency for ensemble mean of different simulation shown in Fig. 1, representing their integrated performance. The SYN_CR begins to deviate from the true values around the early stage of snow accumulation. This is mainly due to overestimation of SCF (due to the error in $\alpha$). During the melting stage, deviation from the true state increases nonlinearly, reaching hundreds of mm. This occurs because errors in SCF parameterization produce a positive feedback in the melting process. A greater SCF results in a higher albedo, causing a decrease in absorbed solar radiation and further increase in SCF. The $\theta$ error further reduces melting amount, because more liquid water is allowed to stay in the snowpack than the true value. In consequence, large error persists in estimated SWE even when observations are assimilated (DA_PAR_CR).

Simultaneously estimating $\alpha$ and SWE with Eq. (1) improves the results of SWE simulation (Fig. 1, DA_PAR_$\alpha$). The innovation (difference between observations and model simulation) in the EnKF updates the parameter ensemble at every time step when observa-

**Table 1**
Description of simulations in experiments.

| | |
|---|---|
| SYN_CR (Section 3) | Synthetic ensemble simulation with parameters error but without synthetic data assimilation |
| DA_PAR_CR (Section 3) | Synthetic ensemble simulation with parameters error and synthetic data assimilation |
| DA_PAR_$\alpha$ (Section 3) | Synthetic ensemble simulation with parameters error, synthetic data assimilation and parameter estimation of $\alpha$ |
| DA_PAR_$\theta$ (Section 3) | Synthetic ensemble simulation with parameters error, synthetic data assimilation and parameter estimation of $\theta$ |
| DA_TRUE (Section 3) | Synthetic ensemble simulation with true parameters and synthetic data assimilation |
| SYN_TRUE (Section 3) | Synthetic truth from a particular forcing set and true parameters |
| DA_STRUCT_TRUE (Section 4) | Synthetic ensemble simulation with parameter error in $\alpha$, other feature same to DA_TRUE |
| DA_STRUCT_NEW (Section 4) | Synthetic ensemble simulation with parameter error in $Z_d$, also with Eq. (6), other feature same to DA_TRUE |

*Note*: In all the tables, SYN refers to "synthetic", CR refers to "control run", DA refers to "data assimilation", PAR refers to "parameter", STRUCT refers to "structure".

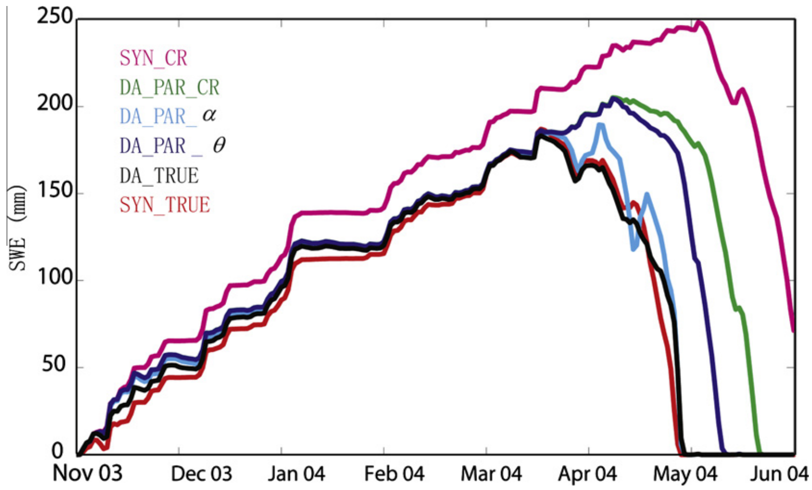


**Fig. 1.** Ensemble mean simulations of SWE (mm) for SYN_CR, DA_PAR_CR and SYN_TRUE, DA_PAR_$\alpha$, DA_PAR_$\theta$, DA_TRUE. (see Table 1 for the description of these simulations).

**Table 2**
The Nush–Sutcliffe efficiency for ensemble mean of different simulation shown in Fig. 1. The value equal to 1 represents the perfect simulation, with the value the larger the better.

| DA_TRUE | DA_PAR_$\alpha$ | DA_PAR_$\theta$ | DA_PAR_CR | SYN_CR |
|---|---|---|---|---|
| 0.98 | 0.96 | 0.61 | 0.22 | −1.21 |

tion is available (every two days). The success of this EnKF in simulating SWE largely depends on an accurate statement of correlations between parameters and observations. This is partly achieved in the representation of energy feedback processes. Similarly, simultaneously estimating $\theta$ and the usual SWE state variable improves the results (Fig. 1, DA_PAR_$\theta$). DA_PAR_$\alpha$ provides better results than DA_PAR_$\theta$, indicating the system's greater sensitivity to errors in $\alpha$. DA_PAR_$\theta$ probably does not fully compensate for bias in related energy processes. When both $\alpha$ and $\theta$ are adjusted the performance is similar to DA_PAR_$\alpha$, and the experiment results are not shown here.

Meanwhile we are interested in that if our method obtains the right results (as shown above) by addressing the right problem. In another word, there should appear "concurrent convergence", i.e., both the state variables and parameters converge to their true values respectively, in the EnKF simulation. Figs. 2 and 3 demonstrate how the mean of parameter ensembles evolves with the sequential EnKF adjustment. True values of $\alpha$ and $\theta$ are approached in the corresponding simulations. Further analyses show that parameter identification depends on a "biased innovation" (which can be interpreted as the difference between the imperfect model run and true state in Fig. 1), and the ensemble estimated correlation between the parameters and observed variables. The "biased innovation" forces the parameter ensembles to vary in a non-stationary mode (changing mean value), while the correlation contributes to controlling the direction and magnitude of that variability. It is noted that estimating both $\alpha$ and $\theta$ leads to good estimates of $\alpha$ and overestimation of $\theta$, for which the ensemble mean remains near 0.06. This suggests competing mechanisms in multi-parameter estimation (especially in a single measurement variable situation), which may favor a few dominant parameters with others not being properly retrieved.

A follow-up question is that whether this EnKF update (note that only SWE and parameters are updated in Eq. (1)) can transfer these benefits to simulation of other snowpack related variables, especially, those related to ground energy balance. Fig. 4 gives the ensemble mean of daily averaged ground radiative temperature for all runs in Table 1, from April 2004 to June 2004 (their results are similar in the accumulation season). In this melting period, as expected, SYN_TRUE and DA_TRUE agree with each other, indicating the ability of data assimilation to accurately estimating the temperature and associated longwave emission of snowpack in the absence of parameter error. And this is achieved

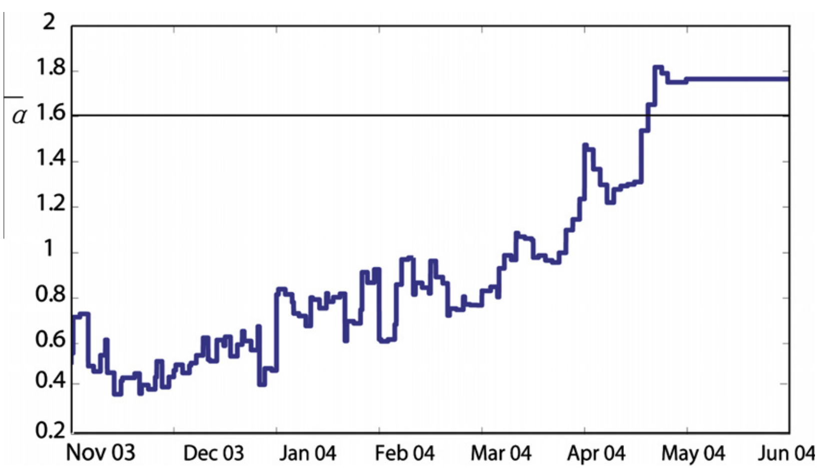


**Fig. 2.** Parameter ensemble mean of $\alpha$ in DA_PAR_$\alpha$.

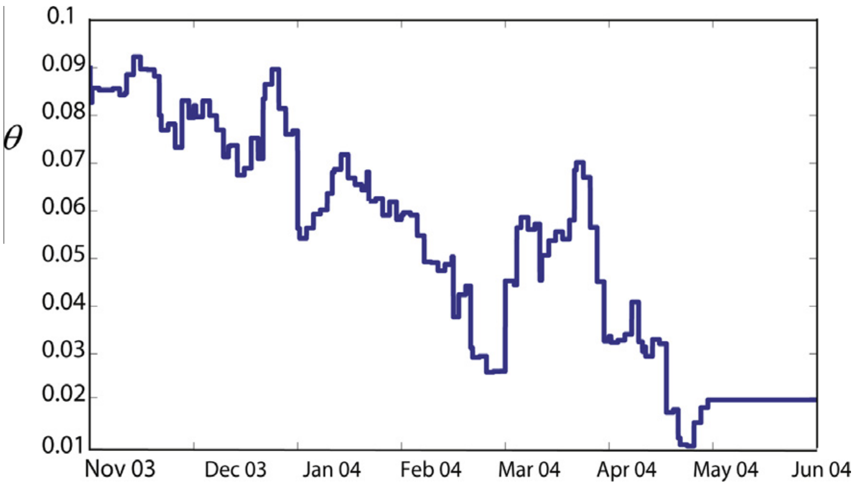


**Fig. 3.** Parameter ensemble mean of $\theta$ in DA_PAR_$\theta$.

indirectly by adjusting SWE, which influence the radiative emission calculation through model physics (better calculation of SCF and absorbed solar radiation in energy balance equation). For SYN_CR and DA_PAR_SYN, the estimation qualities are low. These deficiencies are attributed to the parameter errors that degrade the ground energy balance calculation. The largest gap between these two simulations and the truth appears during the period when ground is snow-free as in SYN_TRUE, implying that whether snow cover is present is a dominant factor to the radiative temperature estimation. For DA_PAR_$\alpha$ and DA_PAR_$\theta$, the simulations are improved by different degrees, with DA_PAR_$\alpha$ much closer to the true value, indicating the stronger influence of $\alpha$ on calculating solar radiative balance.

Figs. 5 and 6 present the simulations of ensemble mean ground sensible heat flux and albedo in daily averaged values. They show similar patterns as compared with Fig. 4. In the melting period, sensible heat flux and grid averaged albedo are closely related to the size of snow area which is linked to SWE in the system. Therefore they can be better estimated if the errors in simulating ground snow cover (for albedo) and radiative energy balance (for sensible heat flux) are resolved. Also, the greater differences in May 2004 (compared to April 2004) between group 1: SYN_TRUE, DA_TRUE, DA_PAR_$\alpha$ and group 2: SYN_CR, DA_PAR_SYN and DA_PAR_$\theta$ result from their different timing of snow-free conditions.

Fig. 7 presents the ensemble mean of diurnal temperature range (DTR) of snowpack. In this experiment the DTR is defined as the temperature (layer averaged) difference between 3 pm and 3 am (local time). This variable can be used to diagnose the thermodynamic effects of liquid water refreezing within snowpack. In this regard the parameter $\theta$ is crucial because it controls the available amount of liquid water to refreeze at night and influence the DTR. The larger $\theta$ leads to more heat loss in a phase change form and decreases the DTR with increased night time snowpack temperature. The results agree with this by showing lower DTR along the entire snow season for simulations with erroneous $\theta$ (SYN_CR, DA_PAR_CR). DA_PAR_$\alpha$ also underestimates DTR, demonstrating the independence between $\alpha$ adjustment and the night time refreezing process. $\alpha$ could also influence DTR by controlling the solar radiation absorption and changing daily maximum temperature. This can explain the significant departure of DTR from the truth in DA_PAR_$\theta$ in the late spring where the SCF decrease.

Adjunct estimations of $\alpha$ and $\theta$ offer results similar to estimating $\alpha$ only, except for DTR calculation, which is similar to adjusting $\theta$ only. They indicate the dominance of $\alpha$ in simulating solar radiative balance, sensible heat flux, and the dominance of $\theta$ in simulating refreezing process of snowpack. The detailed results are omitted.

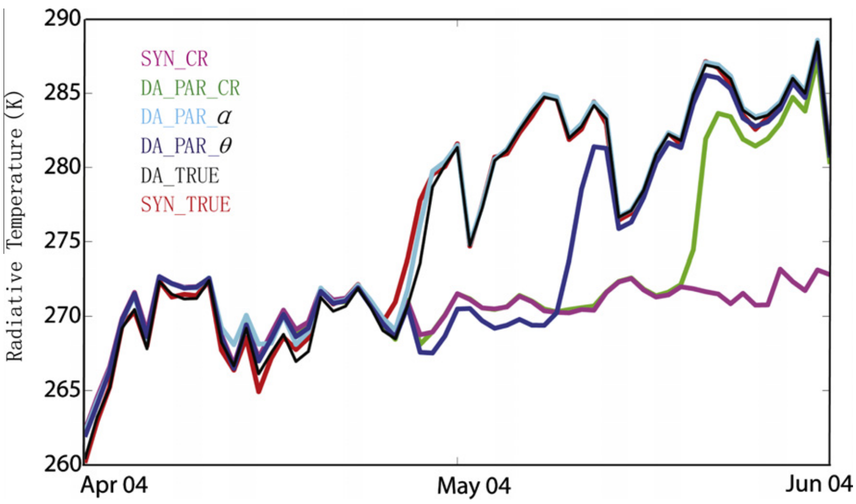


**Fig. 4.** Daily averaged ensemble mean of ground radiative temperature for SYN_CR, DA_PAR_CR and SYN_TRUE, DA_PAR_$\alpha$, DA_PAR_$\theta$, DA_TRUE.

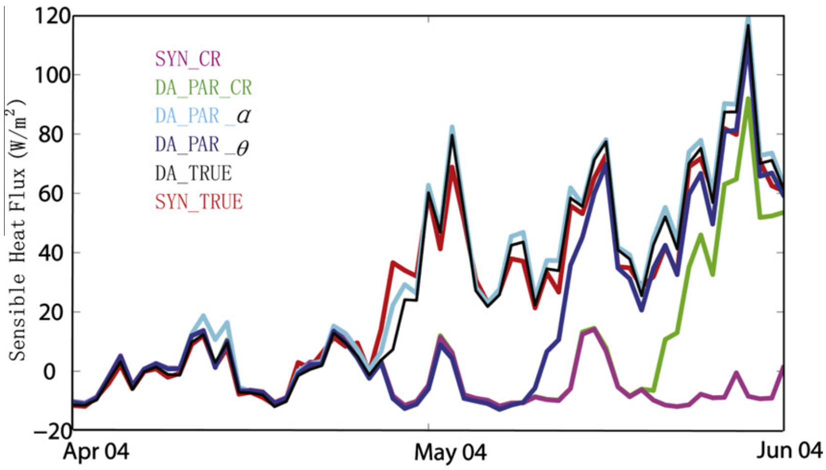


**Fig. 5.** Daily averaged ensemble mean sensible heat flux for SYN_CR, DA_PAR_CR and SYN_TRUE, DA_PAR_$\alpha$, DA_PAR_$\theta$, DA_TRUE.

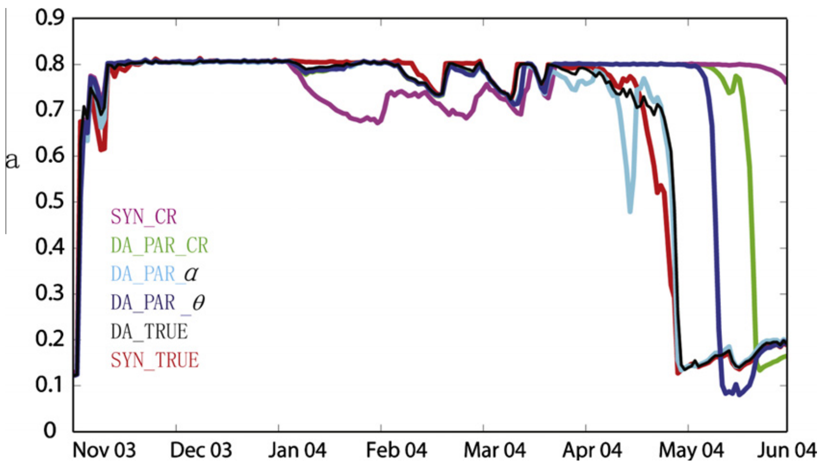


**Fig. 6.** Daily averaged ensemble mean albedo for SYN_CR, DA_PAR_CR and SYN_TRUE, DA_PAR_$\alpha$, DA_PAR_$\theta$, DA_TRUE.

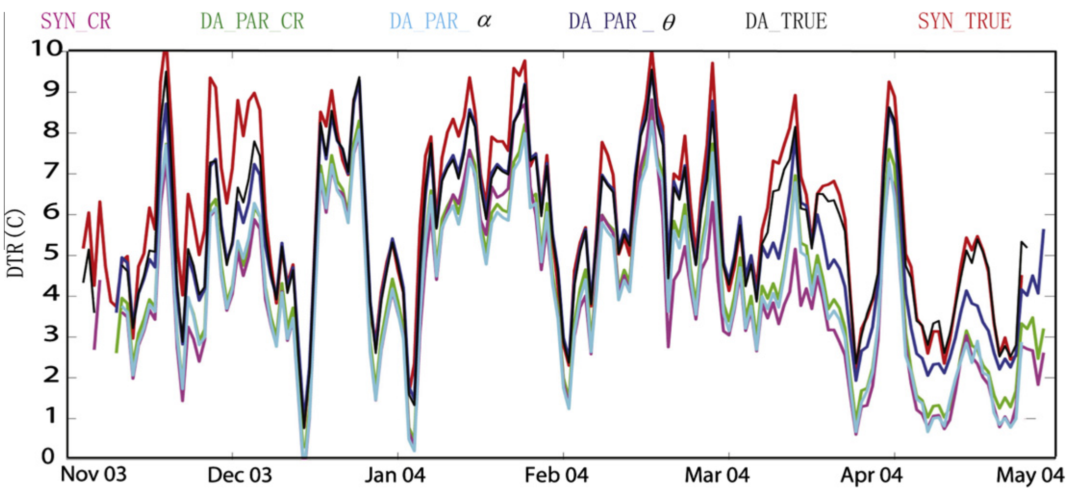


**Fig. 7.** Ensemble mean of DTR (diurnal temperature range) for SYN_CR, DA_PAR_CR and SYN_TRUE, DA_PAR_$\alpha$, DA_PAR_$\theta$, DA_TRUE.

## 4. Effects of structural error on parameter estimation

In this section we only consider a simple case of structural error in snowpack simulation, to facilitate a straightforward diagnose of its effects. Here one primary goal is to keep the diagnostic framework simple and tractable, recognizing that complicated structural error behavior may result from incorporating multiple processes and their nonlinear interaction. Situations consisting of more complex structure errors are valuable topics for further research. Another SCF parameterization (replacing Eq. (3)) is introduced and represents the structural error. It is shown in Eq. (7). In addition to the above mentioned simplicity, at least two reasons are considered to construct the error in this way. First, both two parameterizations share the common role in the model architecture, i.e., representing the same physical process (grid-scale snow cover–snow depth relationship), so they are comparable. Second, they differ greatly in their formulation, representing significant structural difference.

Two simulations are designed. The DA_STRUCT_TRUE is the perfect-model-structure run using Eq. (4), and starts with an erroneous $\theta = 0.6$. The DA_STRUCT_NEW uses Eq. (7) with $Z_d$ as the adjustable parameter. $Z_d$ characterizes the roughness length of ground and is assumed uncertain in DA_STRUCT_NEW. This parameterization was used in the default version of CLM2.0 [5], reflecting the assumption that SCF is lower at uneven ground given same amount of snow. From Eq. (7) it can be easily inferred that SCF is sensitive to $Z_d$ especially in the melting season with modest $h_{sno}$ (for example keeping $h_{sno} = 0.3$ m, $Z_d$ ranging from 0.002 to 0.02 can lead to SCF ranging from 0.93 to 0.6).

To construct a comparable framework, all other structural and parametric features in DA_STRUCT_TRUE and DA_STRUCT_NEW are the same as in SYN_TRUE. Both of them assimilate a common dataset of SWE generated with the same approach in Section 3

$$SCF = \frac{h_{sno}}{10Z_d + h_{sno}} \tag{7}$$

As mentioned in Section 2, the selection of $Q_0$ in Eq. (3) influences the trajectory of parameter ensemble. To ensure a fair comparison, we run the above simulations with different $Q_0$, to obtain enough spread to lead to the dependable results. In addition, the dependence of system performance and parameter retrieval on the selection of initial value of $Z_d$ has been considered. Our preliminary test found that varying this input did not affect the simulation too much, so we use $Z_{d0} = 0.01$ (a default value for CLM2.0, $Z_{d0}$ denotes the initial value of $Z_d$).

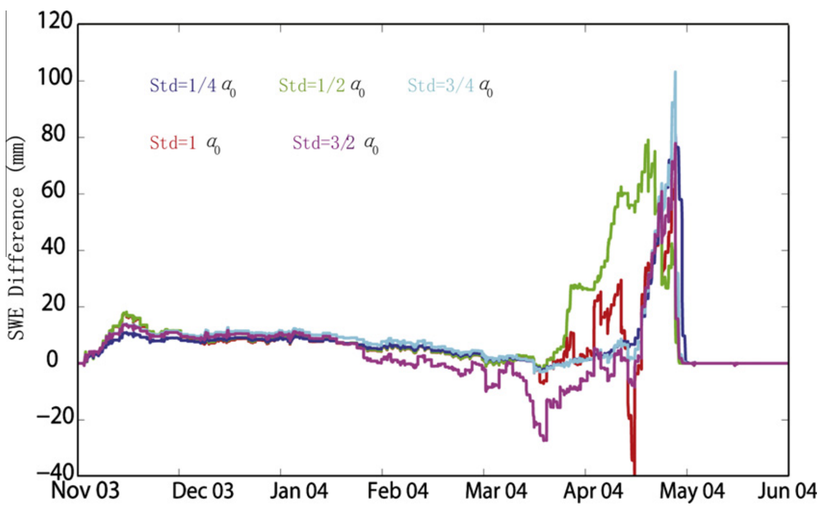


**Fig. 8.** Ensemble mean error in SWE (mean − true value) for DA_STRUCT_TRUE with different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to R multiplying the initial parameter: Std = $R * \alpha_0$.

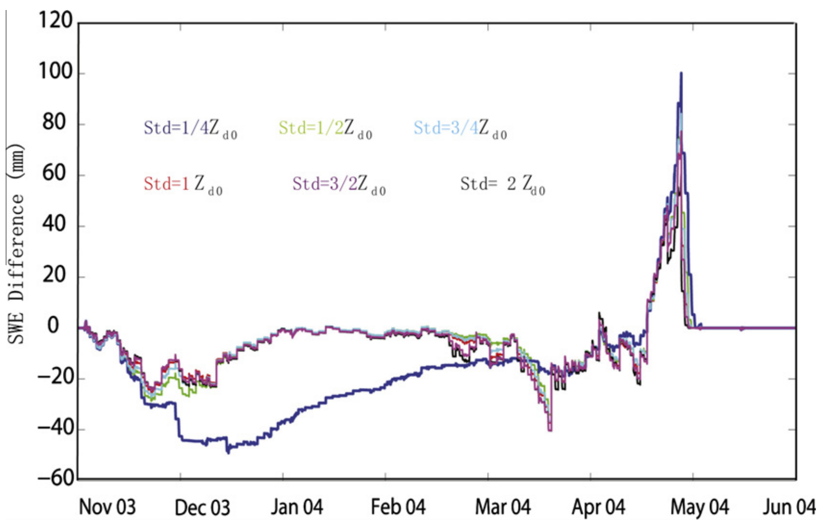


**Fig. 9.** Ensemble mean error in SWE (mean − true value) for DA_STRUCT_NEW with different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * Z_{d0}$.

Figs. 8 and 9 display the error (ensemble mean SWE minus corresponding true value) of DA_STRUCT_TRUE and DA_STRUCT_NEW, respectively, for each of their own $Q_0$ used to constrain the parameter ensemble variance. Note that each curve is given in the same color as the label showing magnitude of square root of $Q_0$ (std). Here std is represented by a number multiplying the initial value. Tables 3 and 4 give the temporal mean of these errors (in absolute value) for DA_STRUCT_TRUE and DA_STRUCT_NEW, respectively. Agreeing with results in Section 3, DA_STRUCT_TRUE performs well in estimating SWE for most of the $Q_0$ selected, except for the std equal to half of $\alpha_0$ (its SWE error is still much lower than the data assimilation run without parameter estimation, not shown here). On the other side, DA_STRUCT_NEW performs almost equally well in estimating SWE (only with significantly larger error for std = $1/4Z_{d0}$, while its SWE error is still much lower than the data assimilation run without parameter estimation, not shown here). Moreover, same pattern exists in both figure, i.e., the error remains low at accumulation period and peaks at melting period. The evolution of parameter ensemble stops when the ground is snow-free for every ensemble member.

Figs. 10 and 11 show how the parameters (ensemble mean) are updated in each simulation. A difference emerges in this comparison: $\bar{\alpha}$ converges at the end of simulation to around 1.7 for different $Q_0$, while $\overline{Z_d}$ diverges to significantly different values for different $Q_0$. Further inclusion of broader range of $Q_0$ in DA_STRUCT_NEW (including std = 1/8 to $3Z_{d0}$, which can give a reasonable performance in SWE estimation) do not result in an evident convergence zone. This has been shown in Tables 5 and 6, where a broad spectrum of $Q_0$ have been tested and the retrieved parameter have been given for DA_STRUCT_TRUE and DA_STRUCT_NEW. These results imply that DA_STRUCT_NEW is likely to reduce the transferability of estimated parameter because no consensus can be made about $\overline{Z_d}$.

**Table 3**
The temporal averaged error in SWE (ensemble mean minus the true value) for parameter estimation run DA_STRUCT_TRUE with different $Q_0$ shown in Fig. 8. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * \alpha_0$.

| $R$ | 1/4 | 1/2 | 3/4 | 1 | 3/2 |
|---|---|---|---|---|---|
| Error (mm) | 7.49 | 11.36 | 8.08 | 7.68 | 7.71 |

**Table 4**
The temporal averaged error in SWE (ensemble mean minus the true value) for parameter estimation run DA_STRUCT_NEW with different $Q_0$ shown in Fig. 9. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * \alpha_0$.

| $R$ | 1/4 | 1/2 | 3/4 | 1 | 3/2 |
|---|---|---|---|---|---|
| Error (mm) | 19.96 | 8.86 | 8.61 | 8.63 | 8.62 |

## 5. Discussion

### 5.1. Parameter estimation convergence/divergence

In a perfect-model-structure run, parameter estimation in the ensemble snow data assimilation has shown promise in accurately estimating a suite of land surface variables. In this scenario, both variables and parameters converge to the true value simultaneously, which is in contrast to the result in the imperfect-model-structure run.

A hypothetical explanation for the above result is that, driven by model structural error, each $Q_0$, a degree of freedom constraining the parameter variance, seems to become an independent condition and leads the stochastic update to a unique value in the parameter space, while to the structure-error free case (DA_STRUCT_TRUE), the effects of $Q_0$ on parameter retrieval appear to be refrained, with different $Q_0$ amounting to largely equivalent constrains, which leads to parameter convergence.

The insensitivity of parameter estimation to the magnitude of prescribed parameter error $Q_0$ in the perfect-model-structure run (in which the forcing and observation error are also perfectly represented) is somehow analogous to results shown in previous research, e.g., Crow and Loon [10], where their experiment demonstrated that, if the single error (only refer to random error, the model structure is perfect) source and observation error are both perfectly represented in the EnKF soil moisture data assimilation system, the overestimation of model error has little impact on the accuracy of retrieved state variable (Fig. 4 in their paper). This linkage implies that the results of parameter convergence (or not) shown here can be interpreted from the observational control perspective, where the structural accuracy might foster a robust relation between observation (invariant for different $Q_0$) and the parameter update. This relationship is important to the availability of a relatively consistent trajectory for the sequential parameter

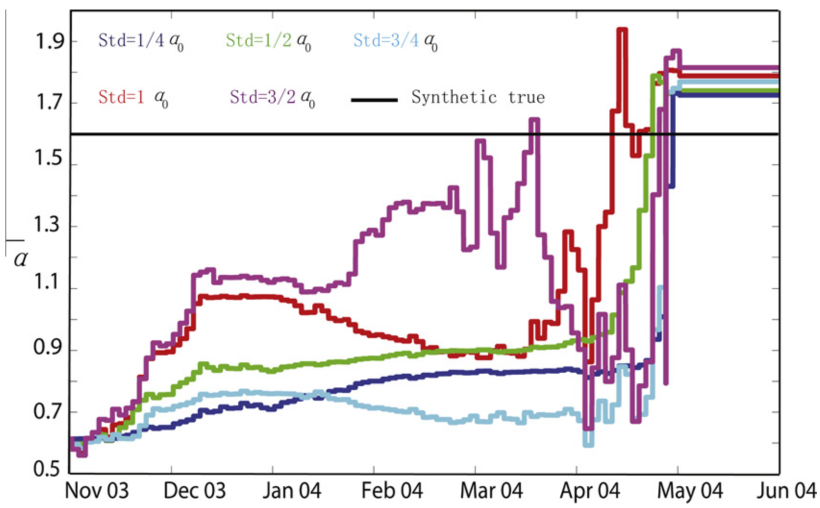


**Fig. 10.** Parameter ensemble mean for DA_STRUCT_NEW with different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * \alpha_0$.

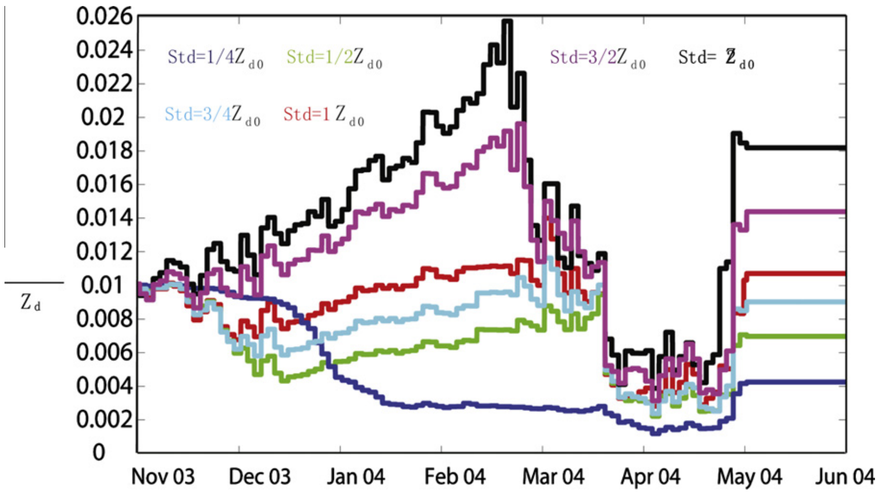


**Fig. 11.** Parameter ensemble mean for DA_STRUCT_NEW with different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * Z_{d0}$.

**Table 5**
The ensemble mean of $\alpha$ retrieved in DA_STRUCT_TRUE when ground is snow-free in May (for every ensemble member), as a function of different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * \alpha_0$.

| $R$ | 1/8 | 1/4 | 1/2 | 3/4 | 1 | 11/4 | 3/2 | 2 | 5/2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{\alpha}$ | 1.70 | 1.72 | 1.73 | 1.77 | 1.78 | 1.79 | 1.81 | 1.83 | 1.89 | 1.96 |

$\frac{\bar{\alpha}_{Max} - \bar{\alpha}_{Min}}{\bar{\alpha}_{Min}} = 0.152.$

search against the variation of $Q_0$. In this experiment, this connection can be partly reflected by the refrained updates of $\alpha$ (in the ablation stage) where $Q_0$ have been increased (Fig. 10, also Table 5, same observation data are used in different simulations). On the other side, the structural error may distort the representation of parameter uncertainty through ensemble and attenuate this connection between observation (structural invariant) and parameter (in the problematic structure), therefore hinder the presence of a consistent parameter evolution among different $Q_0$. This deficiency can be well characterized by an excessively broad divergence zone in the corresponding $Q_0$ space (Fig. 11 and Table 6, same observation data are used in different simulations).

The detailed features associated with the $Q_0$ space may be dependent on a number of complex issues, e.g., the structural role in parameter estimation, which is modulated by the EnKF effects associated with ensemble space dynamics (e.g., using ensemble covariance to calculate increments), also the observation control on parameter evolution. The verifications of the above potential explanation warrant further investigation, which may include how to quantitatively measure these connections and mechanisms (e.g., between the degree of freedom $Q_0$ and parameter evolution), whether they are ad hoc, and what they are conditioned on.

In addition, the above parameter search divergence in DA_STRUCT_NEW could be alleviated by developing a more physically based method to estimate $Q_0$ or predict the variance of parameter ensemble, for example, linking them to the flow-dependent error covariance of state variables (an approach similar to bias estimation in De Lannoy et al. [11]). When $Q_0$ or parameter variance could be calculated through a physically robust scheme, this divergence problem becomes trivial, and the optimal parameter can be identified. However, a fully dynamical parameterization of $Q_0$ can be difficult, considering the nature of parameter (a physically constant value) in the snow hydrological system.

### 5.2. Several limitations in current research

This study may have limitations in several aspects. First, there are alternative approaches to achieve the simultaneous state and parameter estimation that do not require time varying parameter (ensemble) (e.g., [23,8]). The relative strengths and limitations among these algorithms are still not clear, and their optimal design and application are important for further research [20].

We also recognize that the structural error and associated parameter adopted in our research are simplistic. In this regard an extended study with a complex structure error configuration is suggested. Niu and Yang [17] discussed several important physical processes governing snowpack evolution and their various representation in LSM, for example, canopy interception (with or without), radiation transfer (traditional scheme or revised two-stream), and below canopy turbulence. The uncertainties in simulating these processes can be incorporated into current framework as structural error. The interactions among these structural components may give rise to far more complicated results (e.g., different relation between $Q_0$ and parameter search convergence) than those revealed in this work. In addition, because parameters in LSM can have significant and complex interactions ([21]), it is worthy to investigate the appropriate size of parameters involved in this estimation framework, for example, weather to use all snowpack related parameters or to select part of them.

**Table 6**
The $\overline{Z_d}$ retrieved in DA_STRUCT_NEW when ground is snow-free in May (for every ensemble member), as a function of different $Q_0$ constraining the parameter variance. Here $Q_0$ is represented by standard deviation ($\sqrt{Q_0}$), which is equal to $R$ multiplying the initial parameter: Std = $R * Z_{d0}$.

| $R$ | 1/8 | 1/4 | 1/2 | 3/4 | 1 | 11/4 | 3/2 | 2 | 5/2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{Z_d}$ | 0.0026 | 0.0042 | 0.0070 | 0.0090 | 0.0108 | 0.0129 | 0.0142 | 0.0181 | 0.0266 | 0.0362 |

$\frac{\overline{Z}_{dMax} - \overline{Z}_{dMin}}{\overline{Z}_{dMin}} = 12.9.$

## 6. Concluding remarks

This study investigates the performance of parameter estimation in snow data assimilation experiments, and its dependency on the model structure. In the synthetic EnKF simulation without model structure error, simultaneous state and parameter estimation is effective. The algorithm reduces the systematic error in SWE estimates and accurately retrieves the parameter values. Further, a suite of other land surface variables, especially those related to snowpack (and ground) energy balance, are better estimated when the parameters are correctly updated. Another important implication from this research is that, in the presence of model structural error, parameter search convergence and accurate estimation of state variables estimation may not be simultaneously achieved, indicating the potential caveat brought by structure error. In particular, we introduce a new degree of freedom, parameter variance constraint, in the parameter estimation framework, and find that imperfect-model-structure run leads to parameter divergence over a broad zone in this constraint space. This is in contrast to the clearly retrieved parameter convergence in the perfect-model-structure run. These results demonstrate that with a problematic model structure, good performance in estimating state variable does not necessarily reflect that the associated parameter estimation is reliable. In this sense, our investigation may provide a way for diagnosing structural robustness in the ensemble snow data assimilation system. It is also emphasized that the generality of this result should be investigated with a more complex structural error configuration (e.g., a combination of multiple structural components).

## Acknowledgment

This work is funded by the National Aeronautics and Space Administration (NASA) Grants NAG5-10209 and NAG5-12577.

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