Geo. 347k
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THE BEHAVIOR OF LIGHT IN MINERALS AND GEMS: REFRACTION, REFLECTION AND THE CRITICAL ANGLE

The bending of light as it travels through a facet on a gemstone is of great importance to a gemologist because it provides an important means of gem identification (refractive index), and because it is a main factor influencing a gemstones brilliance.

Refractive Index

When light enters a transparent material that has a density that is different from the medium in which it is traveling, its speed changes. Light passing from a less dense medium (e.g. air) to a more dense medium (e.g. gem) is slowed down. Conversely, going from a more dense to less dense medium light speeds up. The **refractive index** (**R.I.**) of a substance is the speed of light within a substance compared to the speed of light in air. For comparison purposes we take the speed of light in air to be equal to 1. Defined in this way, refractive index can be stated as:

$$\text{R.I.} = \frac{\text{velocity of light in air (=1)}}{\text{velocity of light in gem}}$$

It follows from this definition that light in a gem with an R.I. of 2 travels at exactly half its speed in air ($2 = 1/(1/2)$). It also follows that light will travel faster in gems with a low R.I. than in gems with a high R.I.

There is no practical way to measure the velocity of light in most materials, so in order to use R.I. as a means of identification another aspect of the way light behaves upon entering a gem has to be exploited. To measure R.I. we rely on the amount of bending (**=refraction**) of light that occurs when it enters a gem. As a consequence of the change in the speed of light, light rays striking a gems surfaces at any angle other than perpendicular to the surface are bent when entering a gem. Light rays entering a surface perpendicular to the direction of travel <u>are not bent</u>, and pass through the surface in a straight path. Thus the amount of bending that occurs depends not only on the R.I. of the gem but on the angle that the light enters. By convention, all angles for R.I. calculation are defined with respect to an imaginary line, called **the normal**, that is perpendicular to the surface (see Fig. 1). The incoming beam of light, called the **incident ray,** strikes the surface at an angle to the normal called the **angle of incidence**, and is refracted to an angle from the normal called the **angle of refraction**. A simple ratio of the two angles is equal to the refractive index:

$$\text{R.I.} = \frac{\sin\text{(angle of incidence)}}{\sin\text{(angle of refraction)}}$$

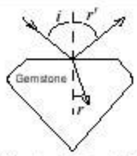

Figure 1. Refracted and reflected light. i = angle of incidence, r = angle of refraction, r' = angle of reflection.

This equation has several features that are important in gemstones. First, it means that if the angle of incidence changes, the angle of refraction must also change because the R.I. must stay the same. How does it change? If the angle of incidence is small (this means that the ray is steeply inclined with respect to the surface, but at a small angle to the normal) then the ray is refracted (bent) less than if the angle of incidence were high. Take a good hard look at Figure 2 that shows this.

Second, because the R.I.s of minerals and gems are always greater than 1 (they range from about 1.4-2.4 for common gem materials), the equation means that the angle of refraction must always be less than the angle of incidence. Stated another way, this means that *light entering a gem will always bend toward the normal and light exiting a gem will always bend away from the normal*.

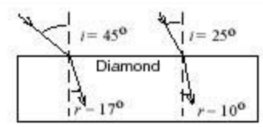

Figure 2. Angle of refraction varies with angle of incidence. Light rays incident upon the surface of a diamond at different angles are refracted differently, but in all cases the ratio sin i: sin r is equal to 2.42, the refractive index of diamond.

Third, the equation means that for a given angle of incidence, *a gem with a higher refractive index will bend light more strongly than a gem with a lower refractive index*. This derives from the fact that when the angle of incidence is held constant, to increase the R.I. the angle of refraction must decrease (look at the equation and prove this to yourself). Remembering that the angle of refraction is defined with respect to the normal, a smaller angle of refraction means a larger amount of bending.

The Critical Angle and Total Reflection

The above discussion of refractive index focuses on how light bends when entering a gem. Brilliance in gemstones reflects primarily how light exits the gem. Light leaving a gemstone either; 1) escapes and is refracted away from the normal or; 2) is completely reflected back into the stone. The most brilliant gemstones are those whose pavilion facets (bottom of the stone) *act like mirrors*, reflecting light back up through the table and crown (top of stone) to the eye. Gemstones whose pavilion facets *behave as windows*, passing light out the bottom of the stone, look dull and less brilliant. What determines whether light inside a gemstone will be internally reflected or pass through a facet? Two things; the angle at which it strikes the facet and the R.I. of the gemstone.

Recall that light leaving a gem will be refracted *away from the normal*, and the amount of refraction depends on the angle at which it strikes the surface and the R.I.. Light striking a facet perpendicular to the facet will escape without being refracted, whereas light striking the facet at a somewhat lower angle will bend away from the normal as it exits (Fig. 3). It should be apparent that there must exist some angle of

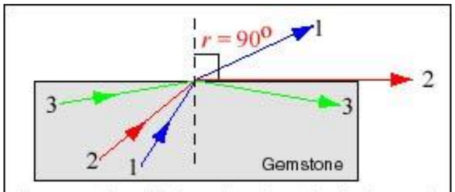

Figure 3. The critical angle and total reflection. The angle between ray 2 and the normal is the **critical angle** for this gemstone because ray 2 is refracted to 90^o from the normal. Ray 1 stikes the gemstone's surface at **less than the critical angle** and exits into air. Ray 3, which strikes the surface at **greater than the critical angle**, is totally reflected and stays within the gemstone.

incidence at which light striking a facet on its way out will be bent far enough to emerge along a path that is exactly parallel to the facet. This angle is a unique property in every transparent mineral and is called the **critical angle** (C.A.). It is related to the refractive index by the simple relation:

$$\sin \text{C.A.} = 1/\text{R.I.}$$

Light striking a facet at an angle greater than the critical angle can not escape through the facet but is instead reflected back into the gem. The equations states that minerals with high R.I.s (e.g. diamond) have low C.A.s and vice-versa.

Critical angle is defined, like the angles of incidence and refraction, with respect to a normal. For quartz, which has a critical angle of 40.2^o, all rays of light exiting facets at this angle to the normal are refracted parallel to the facet they strike. Rays of exiting light striking facets at angles greater than this angle are internally reflected, and bounce around the interior of the stone until they impinge upon a facet at less 40.2^o to the normal, where they then escape. In order to simplify this concept somewhat we could imagine a cone with an opening angle equal to the critical angle of the gem that is centered on a normal to a facet (see Figure 4). These cones are referred to as **cones of acceptance**. All light rays in the gem striking the facet at angles within the cone will escape out the facet, and all light rays striking the facet outside the cone will be reflected off the facet and stay within the gemstone. Gems with a higher R.I.s have *smaller* cones of acceptance (see above), and thus can internally reflect greater amounts of light than gems with lower R.I.s (larger cones of acceptance). **This is the conceptual reason why gemstones of high R.I. materials are more brilliant than those made of materials with a low R.I., all other factors being equal.**

Because gemstones gather light from all angles, there will always be lots of light that will pass through the stone without being internally reflected. Indeed, to try to evaluate all possible ray paths for incoming and exiting light in even the simplest cuts is a daunting if not impossible task. Cuts can be designed by simple, graphical ray tracing of near-vertically incident light entering the table or crown of the stone, and by experimentation, to internally reflect the maximum amount of light back up to the table and crown facets. This is because a large percentage of light striking facets at angles much less than 90^o is reflected from the facet surfaces, never makes it into the stone and thus does not contribute to the brilliance. Although such ray tracing exercises greatly oversimplify the true nature of a cuts brilliance, they reveal certain aspects that lead to several important generalizations that can be applied to all cuts. (For an interesting and enlightening discussion of this topic, see Chapter 7 in Faceting for Amateurs by Glenn and Martha Vargas, on reserve under Geo. 347k in the Geology Library). First and foremost, they reveal that *maximum internal reflection from the pavilion facets occurs only when the culet angle (angle between the girdle and pavilion facets) is larger than the critical angle for the gem*. For example, in the quartz gems which most of you will cut, the culet angle (43°) exceeds the critical angle (40.2°) by nearly 3°. Interestingly, most gems show maximum brilliance for culet angles between 40° and 43°. The greater the difference between the culet angle and the critical angle for the material, the more the gem can be tipped or inclined to the horizontal before the light entering through the table leaks out the botton.

Crown facets act like a lens, refracting exiting light upward toward the viewer. The angle of cut of the crown facets with respect to the girdle is apparently not as critical, but must be less than the critical angle so that light exits and is not internally reflected. For culet angles of 45 °, most light entering the table will be internally reflected and exit though the table, making the crown facets superfilious. *It is thus clear that all culets angles should be less than 45° but greater than the critical angle for the material if the light is to exit through the crown*.

The other role of the crown is as a prism, allowing dispersion to occur for exiting light rays. The flashes of color from the crown of a diamond are an example. Dispersion is only visible if the exiting ray of light is not parallel to the path it takes upon entering the gem.

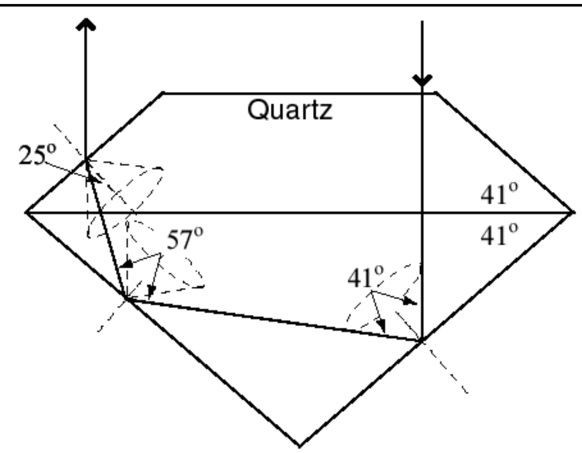

Figure 4. Internal reflection and cones of acceptance for a gem with a critical angle of 40°. Light entering the table strikes the right pavilion facet at 41° to the pavilion normal (dashed). This is greater than the critical angle for quartz (40.2°), as illustrated with the cones, which have opening angles of 40o. The light thus is internally reflected at the angle of incidence (41°) and bounced to the left pavilion facet, where it strikes at 57°. This is also larger thatn the critical angle, thus it bounces again to strike the crown. The angle it strikes the crown (25°) is less than the critical angle, thus the light exits and is refracted.

REFRACTIVE INDEX EXPRESSED BY SPEED OF LIGHT

Substance	**Velocity of Light** miles/second (kilometers/second)	**Refractive Index**
Space	186,282 (299,792)	1.00
Air	186,232 (299,890)	1.00
Water	140,061 (225,442)	1.33
Glass	122,554 (197,349)	1.52
Diamond	77,056 (124,083)	2.42

Refractive Index = Cv /Cs

where Cv is velocity of light in a vacuum (i.e. “Space” above),
and Cs is the velocity in the substance (i.e. “Glass” above).

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SNELLS LAW AND THE CRITICAL ANGLE

Figure 2
Snells Law states:

$$\mathbf{n_r/n_i = \sin i/\sin r}$$

Where:

- $\mathbf{n_r}$ = the refractive index of the medium that light is passing into.
- $\mathbf{n_i}$ = the refractive index of the medium that light is passing out of.
- **i** = the angle that the incident light ray makes with the normal.
- **r** = the angle the light ray is refracted to relative to the normal.

Air has a refractive index of 1.0003, which we will round off to 1. Thus, for <u>light passing from air into a gem</u> (Fig. 1) n_i is 1 and Snells Law simplifies to:

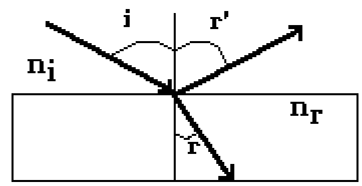

Figure 1.

$$\mathbf{n_r = \sin i/\sin r}$$

For <u>light passing from a gem into air</u> (Fig. 2) the incident ray is within the gem and the medium that light is passing into is air. Thus n_r is 1 and Snells Law is:

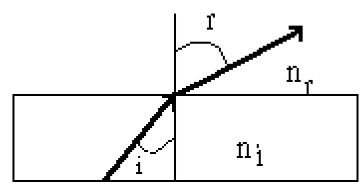

Figure 2.

$$1/n_i = \sin i/\sin r \quad \text{or} \quad \mathbf{n_i = \sin r/\sin i}$$

We define a special angle, the **Critical Angle** (**CA**), as the angle of incidence <u>within a gem</u> for which light is refracted parallel to the surface it is incident upon (Fig. 3). By this definition, the angle of refraction (r) at the critical angle is 90^{o}.
Plugging this special relationship into Snells Law yields the following:

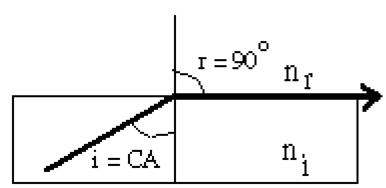

Figure 3.

$$1/n_i = \sin CA/ \sin 90^{o}$$

The sine of 90^{o} equals 1, which reduces the equation to:

$$\mathbf{1/n_i = \sin CA}$$

This equation simply states that high refractive index materials have small critical angles and vise versa.