Geodesy, Geographic Datums & Coordinate Systems

- What is the shape of the earth?
- Why is it relevant for GIS?
From Conceptual to Pragmatic

- Dividing a sphere into a stack of pancakes (latitude) and segments of an orange (longitude) is useful for navigation (relative to Polaris) and keeping time on a rotating sphere (15° long. = 1/24 of a rotation = 1 hr).
- How can we make graphs (= paper or digital maps) in Cartesian units (e.g. meters, feet) relative to this concept?

CONVERT DEGREES TO OTHER UNITS

e.g. How many degrees are in a meter of Latitude or Longitude?
Map-making of Places on Earth Involves Two Conceptually Steps:

1. Make an accurate 3D model of earth – e.g. an accurately scaled globe – to establish horizontal and vertical *measurement datums* - **TODAY**

2. Flatten all or part of that globe to a 2D map (via. a projection technique) and define a Cartesian coordinate system – **NEXT TIME**

![Map-making diagram](attachment:image.png)

- **Scale**: 1: 42,000,000
- **Scale Factor (for areas)**: 0.9996

*Peters Projection*
Make a Map, Graph the World

- What determines spacing of 30° increments of Lat. & Lon.?
- **Dimensions and shape (“figure”) of earth**
  - Model vs. Reality
  - Measurement Accuracy

*Graph shows 30° increments of Lat. & Lon.*

*Austin: (-97.75, 30.30)*
The “Figure” of the Earth

- **Reference Models**
  - **Sphere** with radius of ~6378 km
  - **Ellipsoid** (or Spheroid) with equatorial radius (semimajor axis) of ~6378 km and polar radius (semiminor axis) of ~6357 km
    - Difference of ~21 km usually expressed as “flattening” \( f \) ratio of the ellipsoid:
      - \( f = \frac{\text{difference}}{\text{major axis}} = \frac{1}{300} \) for earth
      - Expressed also as “inverse flattening”, i.e. 300

- *(Geodesy is the science of measuring the size and shape of Earth and locations of points on its surface)*
Model Ellipsoid of Revolution/Spheroid

- Rotate an ellipse around a vertical axis (c.f. Oblate indicatrix of optical mineralogy)

  \[ a = \text{Semimajor axis} \]
  \[ b = \text{Semiminor axis} \]
  \[ X, Y, Z = \text{Reference frame} \]
  \[ f = \frac{a - b}{a} = \text{“flattening”} \]
  \[ \frac{1}{f} = \frac{a}{a - b} = \text{“inverse flattening”} \]
Two (of many) Standard Earth Reference Ellipsoids:

<table>
<thead>
<tr>
<th>Ellipsoid</th>
<th>Major Axis $a$ (km)</th>
<th>Minor Axis $b$ (km)</th>
<th>Inverse Flattening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clark (1866)</td>
<td>6,378.206</td>
<td>6,356.584</td>
<td>294.98</td>
</tr>
<tr>
<td>GRS 80</td>
<td>6,378.137</td>
<td>6,356.752</td>
<td>298.257</td>
</tr>
</tbody>
</table>

- At least 40 other ellipsoids in use globally
And The Answer Is:

<table>
<thead>
<tr>
<th>Ellipsoid</th>
<th>$1^0$ of Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clark (1866)</td>
<td>~110,591 meters</td>
</tr>
<tr>
<td>GRS 80</td>
<td>~110,598 meters</td>
</tr>
</tbody>
</table>

~ 7 meter difference is significant with modern software, but the real difference is the Datums with which they are typically associated.
Horizontal Reference Datums

Horizontal Datum = 1) shape and size of reference ellipsoid AND 
2) location of ellipsoid center relative to center of mass of earth 
(geocenter).

Common North American datums:
- **NAD27** (1927 North American Datum)
  - Clarke (1866) ellipsoid, *non-geocentric* (local) origin*
- **NAD83** (1983 North American Datum)
  - GRS80 ellipsoid, *geocentric* origin for axis of rotation
- **WGS84** (1984 World Geodetic System)
  - WGS84 ellipsoid; geocentric, nearly identical to NAD83
- [Other datums](#) in use globally
Datums and the Geocenter

❖ Geocenter = center of mass of earth

❖ Local Datum vs. Geocentric Datum

Local Datum, e.g. NAD27
Point of tangency

Geocenter

Earth’s Surface

WGS84 datum

Geocentric Datum e.g. WGS84 or NAD83

NAD27 datum
National Geodetic Survey (NGS)  
“Geodetic Datum”

- A set of constants specifying the coordinate system used for geodetic control; a fitted reference surface, e.g. NAD83(1986)
- Surface based on precisely determined coordinates for a set of points - “benchmarks” - empirically derived from astronomical, satellite and distance measurements
- Used for calculating the coordinates of points on Earth
- NAD83 is the modern (legal) horizontal geodetic datum for US, Canada, Mexico and Central America
- Different versions, e.g. NAD83(1996), NAD83(2011) are different “realizations”, refinements
Adjustments to NAD83

- **HARN (or HPGN)** – High Accuracy Reference Network = *Empirical corrections to NAD83(1986)*

- **Cooperative initiative between N.G.S. and states using GPS** to refine NAD83 network of control points

- Network of ~16,000 stations surveyed from 1989-2004, allowing network accuracy of 5mm for state NAD83(HARNs)

- Subsequent refinements based on ~70,000 GPS stations: NAD83(CORSxx), NAD83(2011)
World Geodetic System 1984-WGS84-Datum

- Devised by Department of Defense for global use
- Introduced in 1987
- Uses WGS84 ellipsoid (=GRS80)
- Several “realizations”, e.g. WGS84(G873), WGS84(G1150), all yielding slightly (<1m) different locations for points
- Commonly the default datum for GPS instruments
- Equating to NAD83 without conversion can yield up to 2m errors.
Datum “shifts”

- Coordinate shift by application of wrong datum can result in horizontal positioning errors as great as 800 m.
- An example compares the WGS84 location of the Texas state capitol dome to 13 other datums.
- Largest (<200m) U.S. shifts typically from misapplying NAD27 to NAD83 data or vice-versa.
- Shifts of ≤2 meter common for different realizations of NAD83; up to 2 meters for WGS84 vs. NAD83.
NAD27, NAD83 & WGS 84 Coordinates

<table>
<thead>
<tr>
<th>Datum</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>30.283678</td>
<td>-97.732654</td>
</tr>
<tr>
<td>NAD83</td>
<td>30.283658</td>
<td>-97.732548</td>
</tr>
<tr>
<td>WGS84</td>
<td>30.283658</td>
<td>-97.732548</td>
</tr>
</tbody>
</table>

(Bellingham, WA)
Datum Transformations - Theoretical

- Equations relating Lat. & Lon. in one datum to the same in another:

  Convert Lat., Lon. and elevation to X, Y, Z

  - Using known X, Y, Z offsets of datums, transform from X, Y, Z of old to X, Y, Z of new
  - Convert new X, Y, Z to Lat., Lon. and elevation of new datum
  - E.g. Molodensky, Helmert Geocentric Translations
Datum Transformations - Empirical

Use Grid of differences to convert values directly from one datum to another. Best for converting between old and new datums.

- E.g. NADCON (US), NTv2 (Canada)
- Empirical; potentially most accurate (NAD27 to NAD83 accurate to ~0.15 m for Cont. US)
- HARN and HPGN values used for grid to update NAD83
  - Stand-alone programs are available to do conversions by most methods; also done within ArcGIS ArcMap & Toolbox
  - See Digital Book on Map Projections for more info.
Latitude and Longitude

- Historical Development
- Coordinates on an ellipsoidal earth

+30° (North) Latitude
-30° (West) Longitude
Coordinates Have Roots in Maritime Navigation

- **Latitude**: measured by vertical angle to polaris (N. Hemisphere) or to other stars and constellations (S. Hemisphere)
- **Longitude**: determined by local time of day vs. standard time (e.g. GMT)
  - requires accurate clocks; 1 hour difference = $15^\circ$ of Longitude*
Latitude(\(\phi\)) on Ellipsoidal Earth

Latitude of point U calculated by:

1) Defining the tangent plane (\(fg\)) to the ellipsoid at U.

2) Defining the line perpendicular to the tangent plane (\(cd\)) passing through U.

3) Latitude (\(\phi\)) is the angle that the perpendicular in 2) makes with the equatorial plane (angle \(cde\)).
Latitude facts:

- Lines of latitude (parallels) are evenly spaced (“small circles”) from $0^\circ$ at equator (a “great circle”) to $90^\circ$ at poles.
- 60 nautical miles ($\sim 110$ km)/$1^\circ$, $\sim 1.8$ km/minute and $\sim 30$ m/second of latitude.
- N. latitudes are positive ($+\phi$), S. latitudes are negative ($-\phi$).
Longitude ($\lambda$)

Longitude is the angle ($\lambda$) between the plane of the prime meridian and the meridional plane containing the point of interest (P).
Longitude facts:

- Lines of longitude (meridians) converge at the poles; the distance of a degree of longitude varies with latitude.
- Zero longitude is usually the Prime (Greenwich) Meridian (PM); longitude is measured from 0-180° east and west of the PM (or other principal meridian).
- East longitudes are positive (+λ), west longitudes are negative (-λ).
Units of Measure

- Decimal degrees (DD), e.g. - 90.50°, 35.40°
  - order by long., then lat.
  - Format used by ArcGIS software
- Degrees, Minutes, Seconds (DMS), e.g. – 90° 30’ 00”", 35° 24’, 00”
- Degrees, Decimal Minutes (DDM) e.g.
  – 90° 30.0’, 35° 24.0’
Vertical Datums

- Mean Sea Level (MSL) – historical datum only, not level!
- Geoid (datum for “Orthometric” Height)
  - Geoid = surface of constant gravitational potential that best fits MSL
    - governed by mass distribution of earth
    - shape is empirically (measurement) based – not a geometrical model
    - datum that most closely approaches historical MSL
- Ellipsoid (datum for Height above ellipsoid: HAE)
  - Geometrically simple (“level”) surface
  - Datum used by most GPS receivers
Can’t directly observe Geoid or Ellipsoid

So traditionally MSL heights found by level line surveys away from coasts.

- Use plumb bob to establish horizontal
- Use optical instruments and trigonometric relationships
Sea Level (MSL), Geoid

- Measure gravity (via satellites) and connect with tide gauge(s) on land to “calibrate” geoid to elevation. Set to zero, or more commonly to nonzero historical match.

- Sea “Level” (geoid) not level; as much as 85 to -105 m of relief globally.
Geoid, Ellipsoid and Elevation (H)

\[ h = H + N \quad \text{or} \quad H = h - N \]
GEOID99 heights (= Geoid – Ellipsoid) range from a low of -50.97 m (magenta) in the Atlantic Ocean to a high of 3.23 m (red) in the Labrador Strait.

Geoid of the World (EGM96)

“Potsdam Gravity Potato” (Geoid 2011)
from GRACE satellite measurements
To convert HAE to Orthometric (elev. above MSL) Height:

- Need accurate model of geoid height (e.g. N.G.S. GEOID99)
  - GEOID99 has 1 x 1 minute grid spacing
- Compute difference between HAE and Geoid height ([online here](#) for US)
- Current model allows conversions accurate to ~ 5 cm
- More precise orthometric heights require local gravity survey
North American Vertical Datums

- National Geodetic Vertical Datum 1929 (NGVD29)
  - ~Mean sea level height based on 26 tide gauges and 1000’s of bench marks. Not MSL, *not Geoid, not an equipotential surface*
  - Failed to account for sea surface topography (unknown at the time)

- North American Vertical Datum 1988 (NAVD88)
  - Latest, established 1991
  - Fixed to 1 tidal benchmark in Quebec
  - Based on best fit to vertical obs. of US, Canada and Mexico benchmarks
Next time: How do we get from 3D earth models to 2D maps?

- Map Projections – transforming a curved surface to a flat graph
- Rectangular coordinate systems for smaller regions – UTM, SPCS, PLS