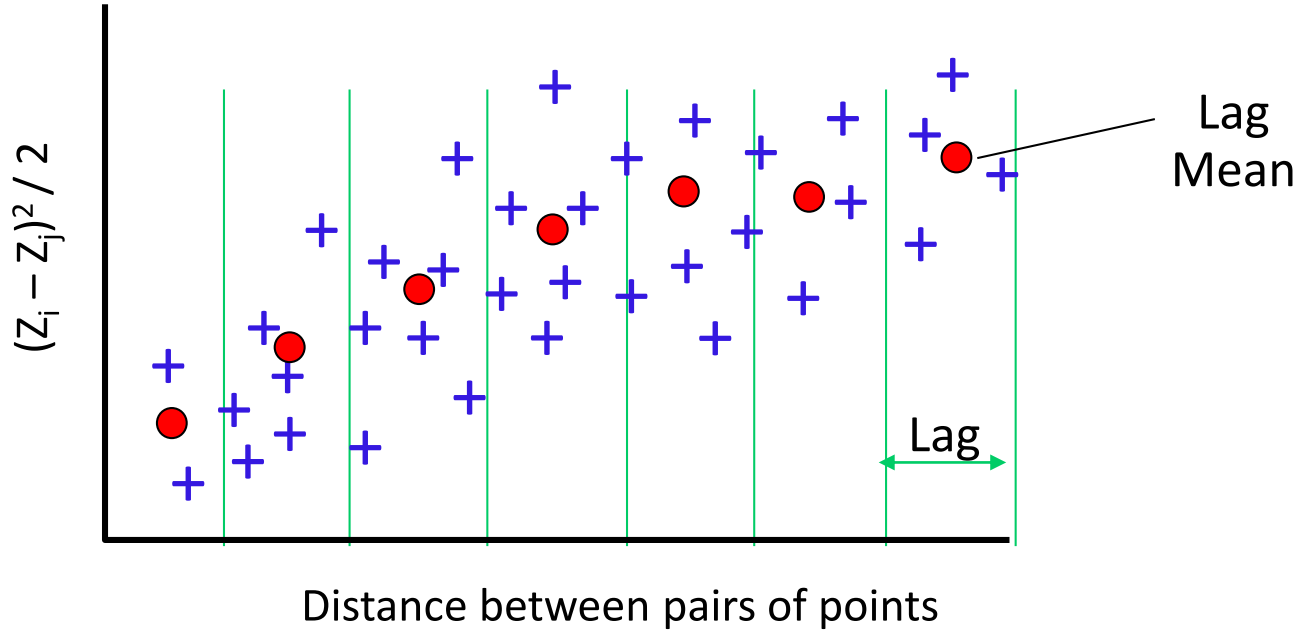
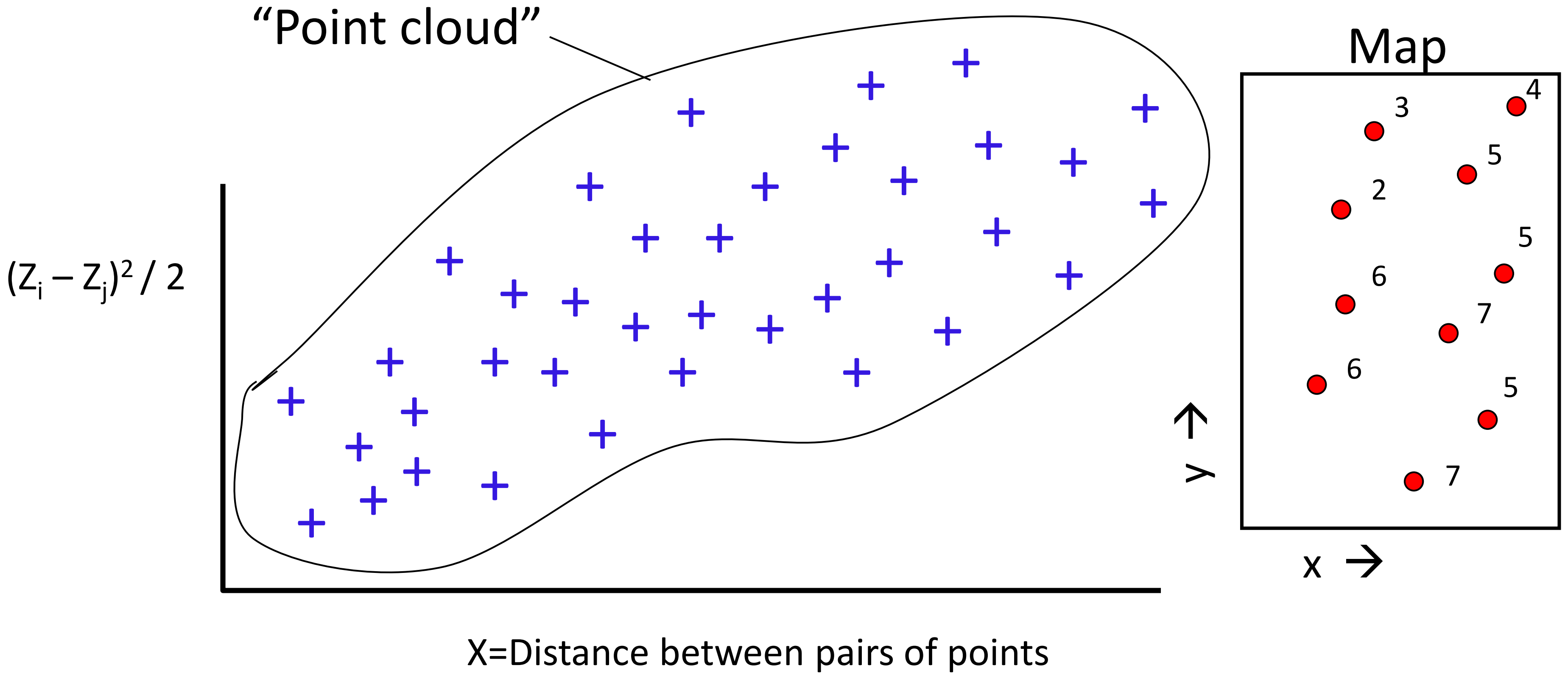
Spatial Interpolation & Geostatistics



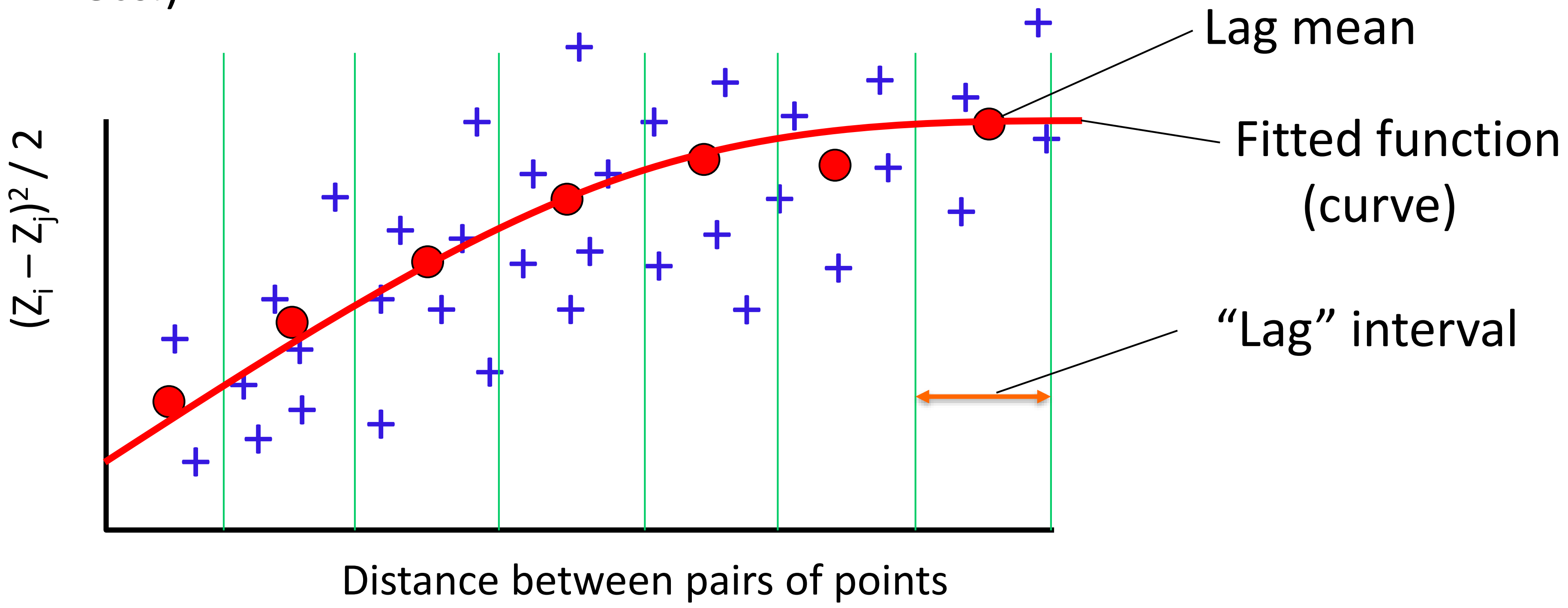
Kriging – Step 1

- Describe spatial variation with *Semivariogram*



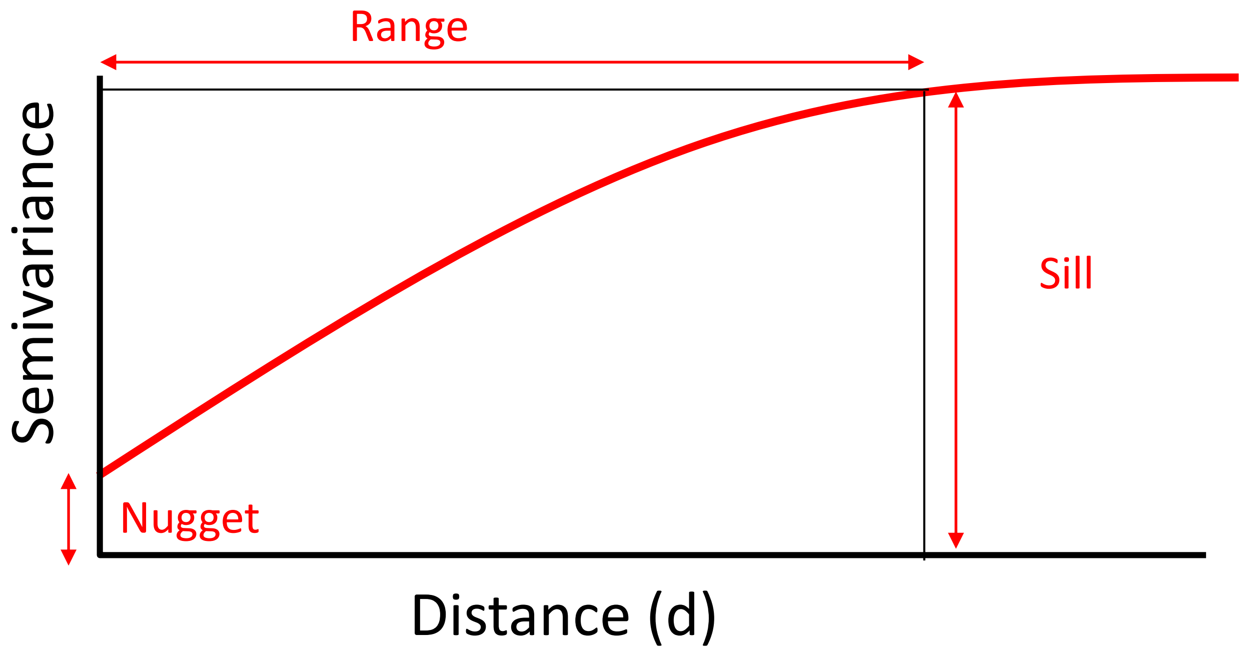
Kriging – Step 2

- Summarize spatial variation with a function
  - Several choices possible; curve fitting defines different types of Kriging (circular, spherical, exponential, gaussian, etc.)



Kriging – Step 2

- Key features of fitted variogram:



Nugget: semivariance at d = 0

Range: d at which semivariance is constant

Sill: constant semivariance beyond the range

Kriging – Part II

- ***Goal:*** predict values where no data have been collected
  - Relies on first establishing:
    - DEPENDENCY – z is, in fact, correlated with distance, i.e. Tobler's Law
    - STATIONARITY – z values are stochastic (except for spatial dependency they are randomly distributed) and have no other dependence – use "detrending" or transformation tools if not Gaussian
    - DISTRIBUTION – works best if data are Gaussian. If not, they have to first be made close to Gaussian.

ESRI Geostatistical Analyst Products

- Map types:
  - Prediction – contours of interpolated values
  - Prediction Standard Errors – show error distribution, as quantified by minimized RMS error (see below)
  - Probability – show probability of values exceeding a specified threshold
  - Quantile – show where thresholds overestimate or underestimated predictions

ESRI Geostatistical Analyst Products

E.g. Maps of Maximum Ozone Concentrations, 1999

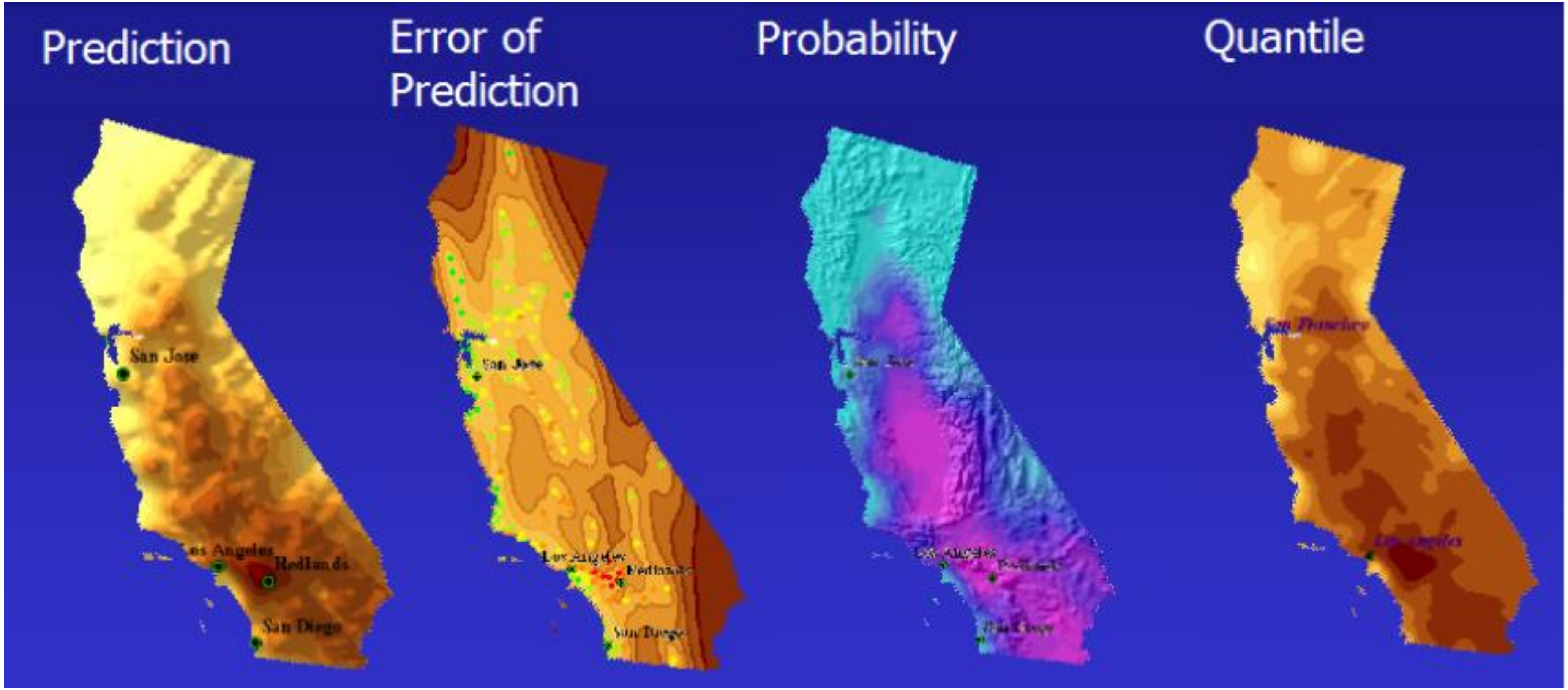
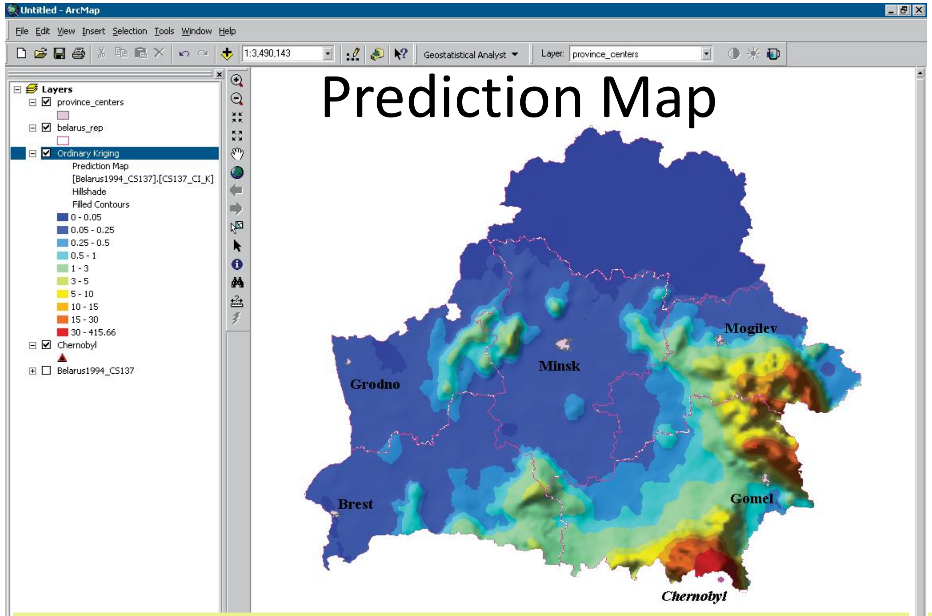


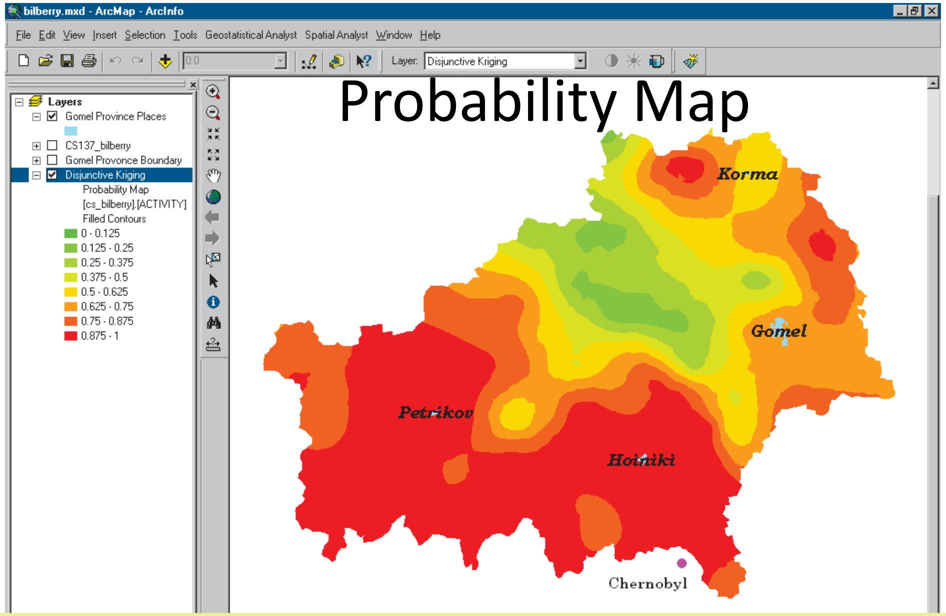
Figure from ESRI "Intro. to Modeling Spatial Processes Using Geostatistical Analyst"

Some Kriging Products

- Prediction map – interpolated values – like produced by other interpolation techniques
- Probability map- showing where critical values exceeded



Prediction map of radioceasium soil contamination levels in of Belarus after the Chernobyl nuclear power plant accident.



Dark orange and red indicate a probability greater than 62.5% that radioceasium contamination exceeds the critical threshold in forest berries.

Figures from ESRI "Using Geostatistical Analyst"

Kriging – Part II

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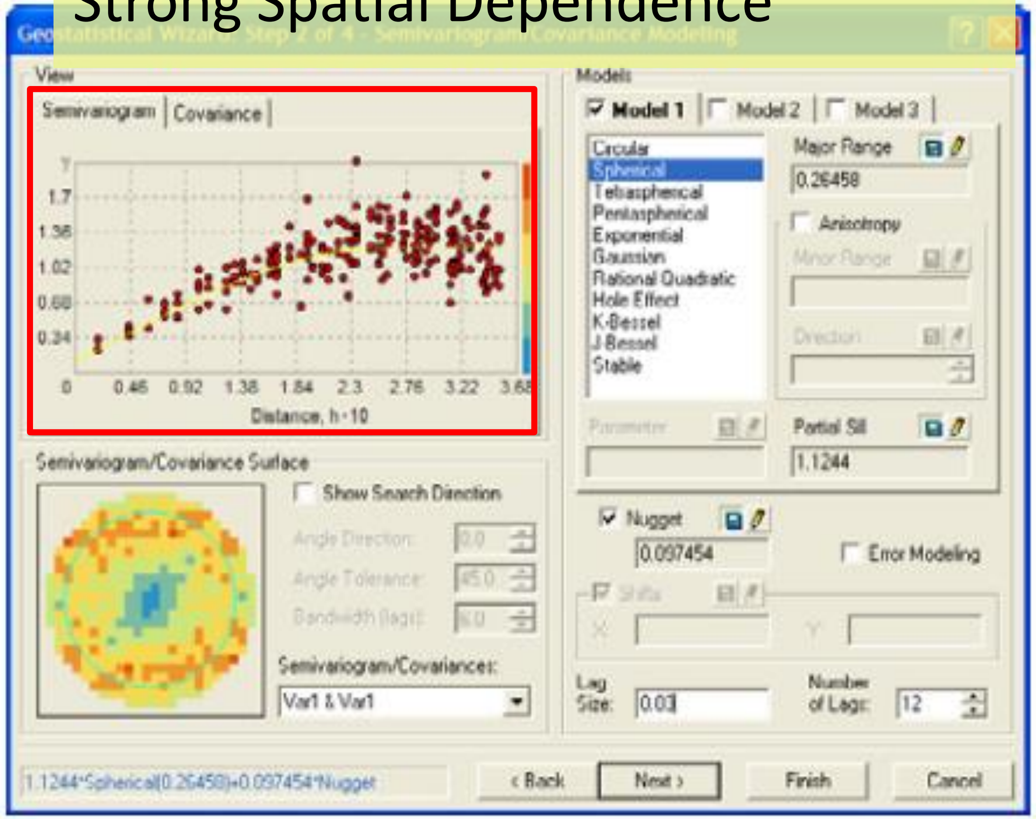
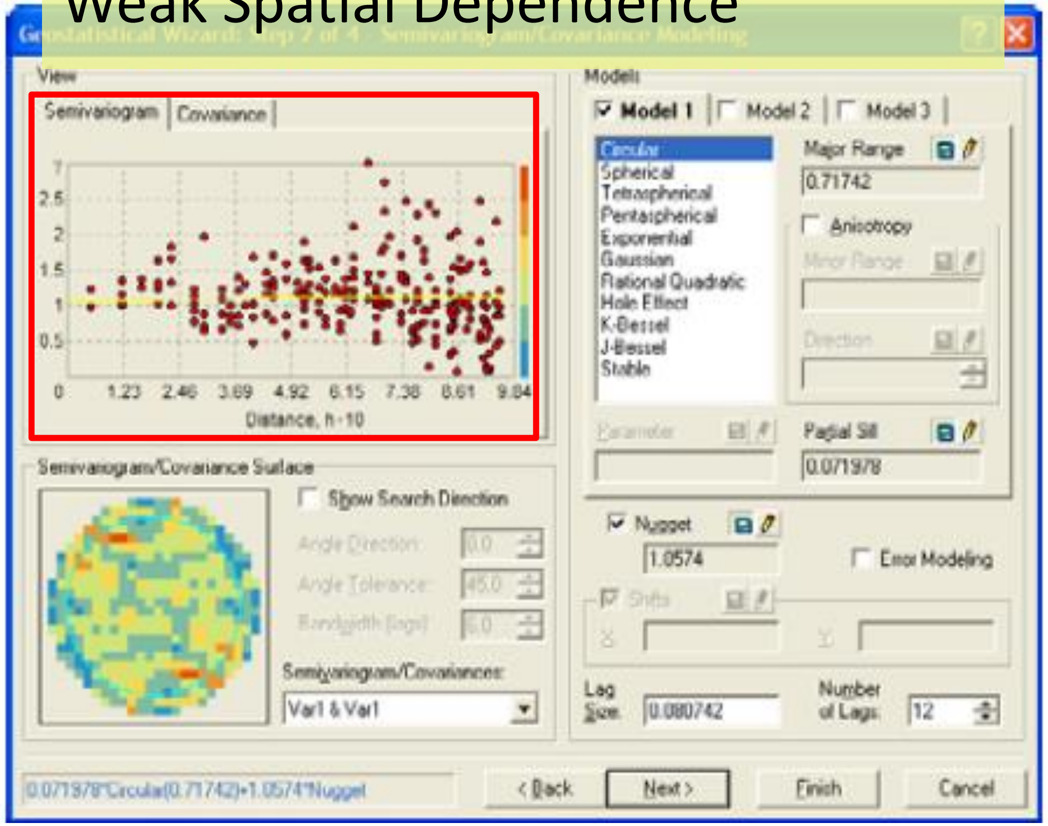
Spatial Dependence:
Semivariogram and Semivariogram Surface

Strong Spatial Dependence



Weak Spatial Dependence

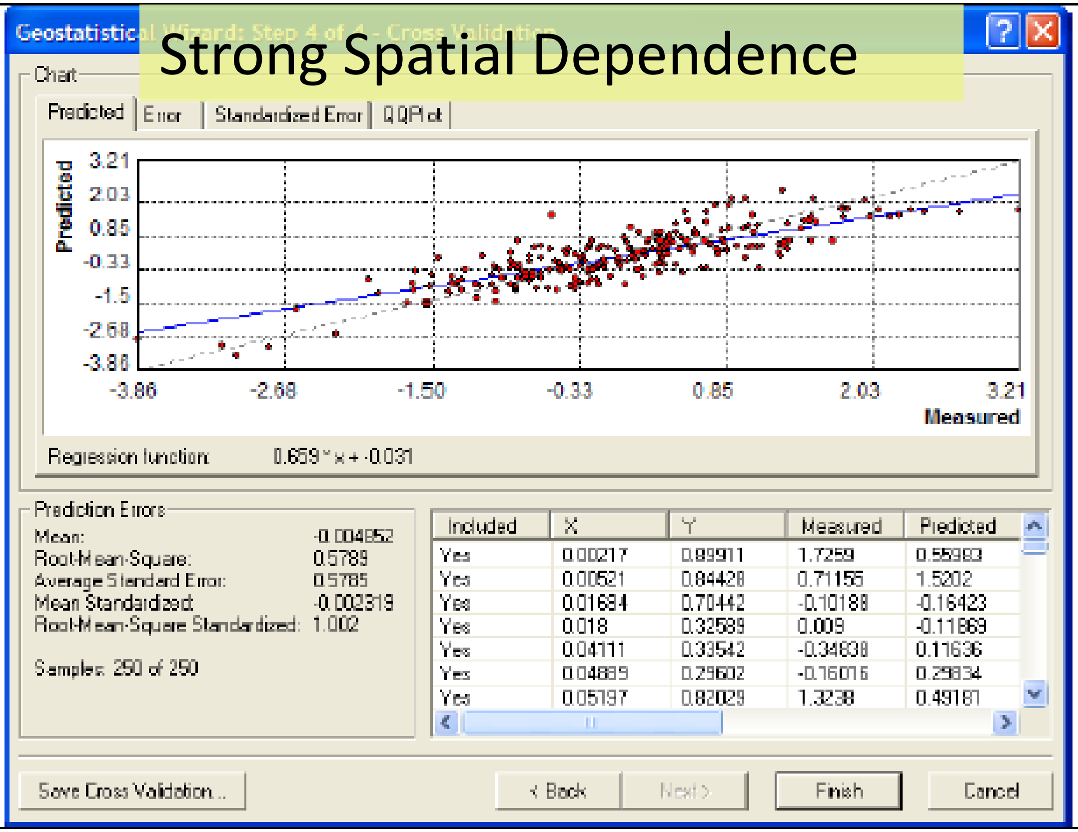


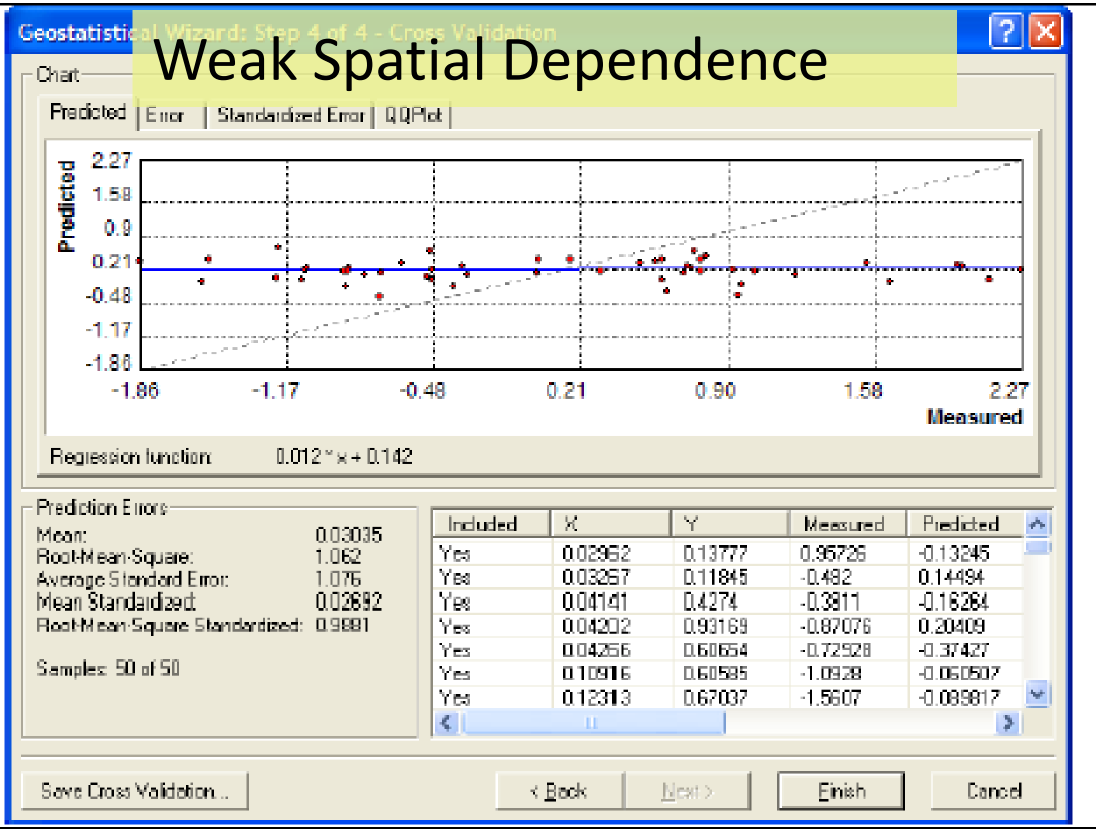
Figures from ESRI “Intro. to Modeling Spatial Processes Using Geostatistical Analyst”

Spatial Dependence:

Cross-Validation Diagnostic – Predicted vs. Measured

- Use a **subset of the data** to test measured vs. predicted values





Figures from ESRI "Intro. to Modeling Spatial Processes Using Geostatistical Analyst"

Kriging – Part II

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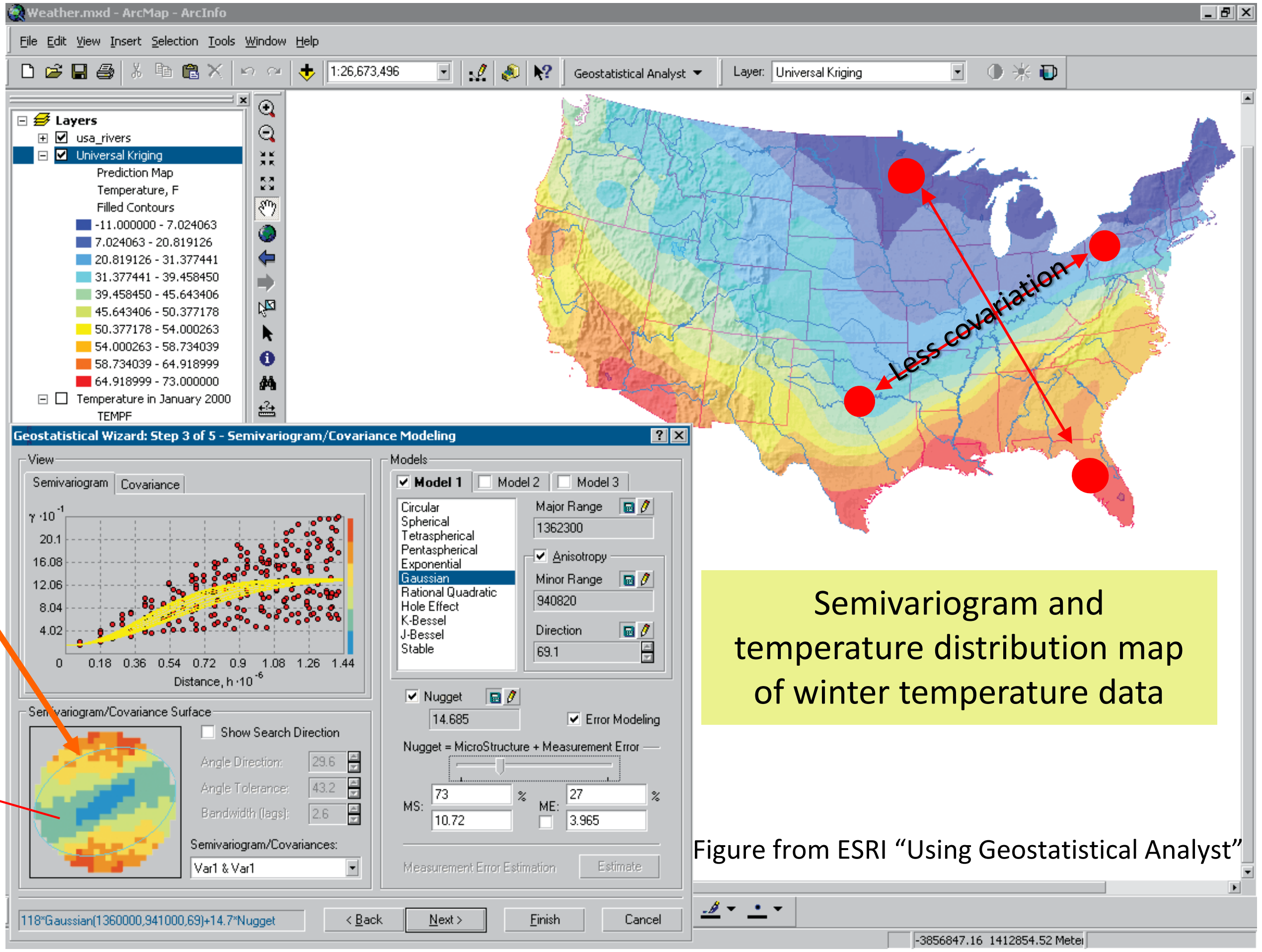
2. Stationarity

- Test with semivariogram & cross-validation plots

Violates Stationarity; detrend first

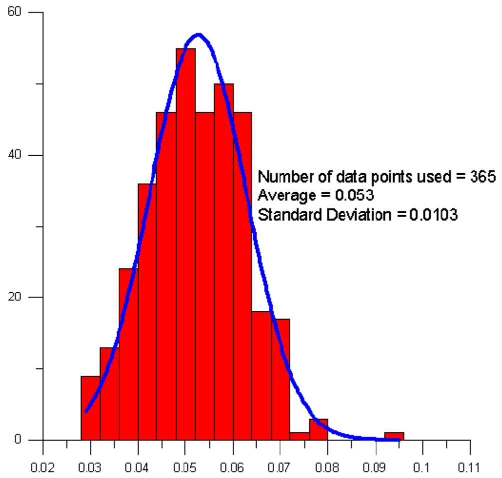


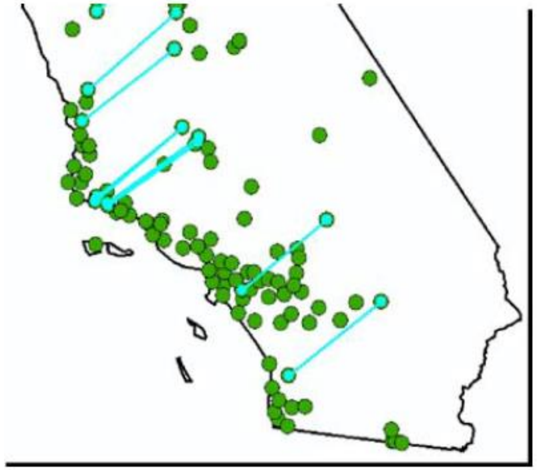
Covariance Surface, Cool!

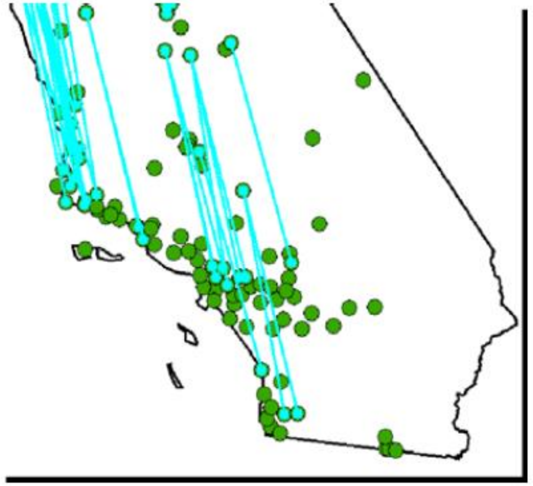
stationarity is the assumption that the covariance between two points is the same for a given distance *and direction*, regardless of which two points are chosen.

2. STATIONARITY - Randomness

- Data variance and mean is the same at all localities (or within a neighborhood of nearest points); data variance is constant in the neighborhood of investigation
- Correlation (covariance) depends only on the vector that separates localities, not exact locations, number of measurement or direction (anisotropy)







Figures from ESRI “Intro. to Modeling Spatial Processes Using Geostatistical Analyst”

California Ozone Demo.

- Data in "Geostat_demo" zip folder in Canvas Module for Week 10

ArcGIS Kriging Processing Steps

1. Add and display the data
2. Explore the data's statistical properties
3. Select a model to create a surface – make a prediction map!
4. Assess the result
5. Compare to other models

Data Exploration

1. Examine the distribution – normal (Gaussian)? Transformation to normal required?
   - *Histograms and QQ Plots*
2. Identify trends, if any
   - *Trend Analysis*
3. Understand spatial autocorrelation and directional influences

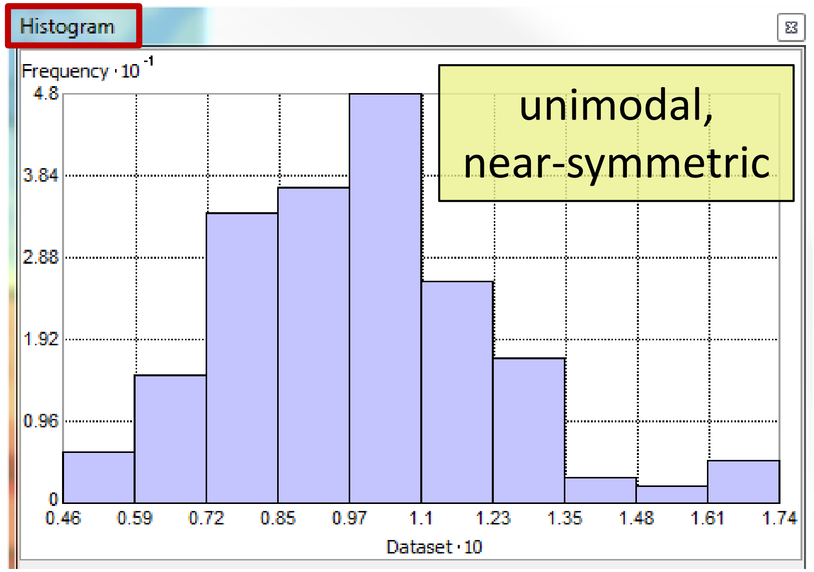
   Semivariogram analysis

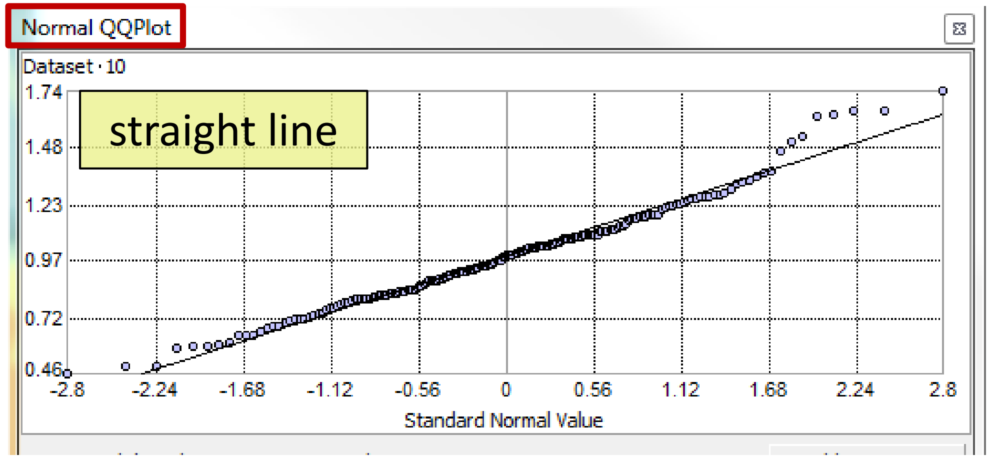
Data Exploration:
Examine the Distribution

- Normal (Gaussian) distribution? (central value, spread, symmetry; mean and median the same?) Standard Normal: mean = 0, standard dev. =1

  Transformation to normal required?

  - Histogram tool, QQPlot tool (compare real and standard normal distributions)

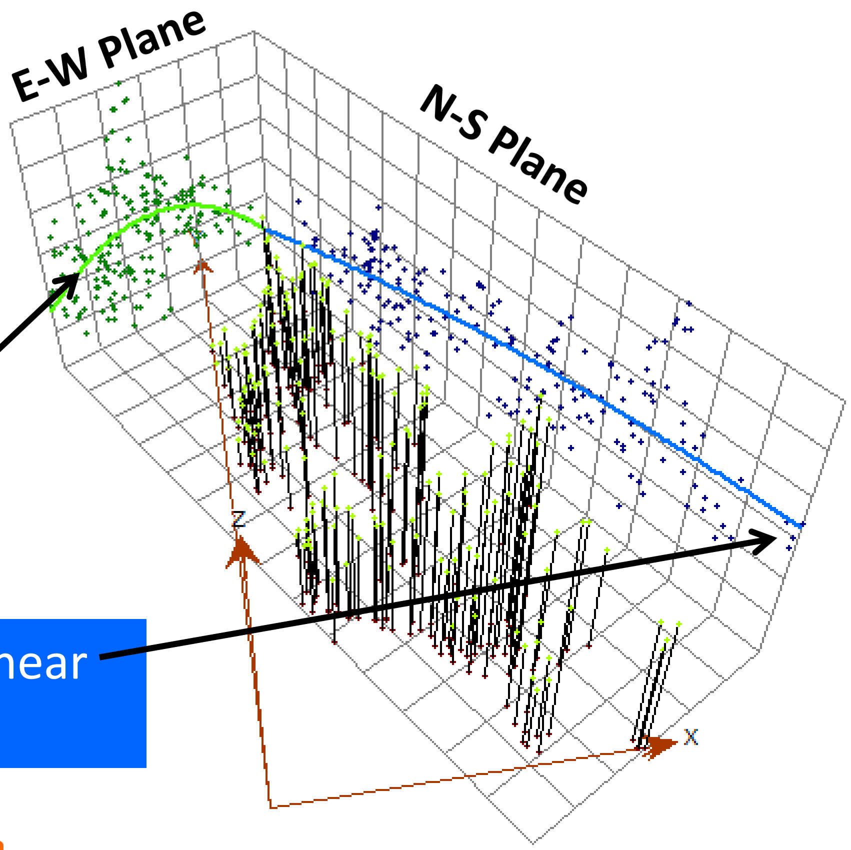




= nearly Normally Distributed data

Data Exploration:
Identify Trends, If Any

- Underlying trends affect Kriging assumption of randomness – remove and work with "residuals"
  - Trend Analysis tool

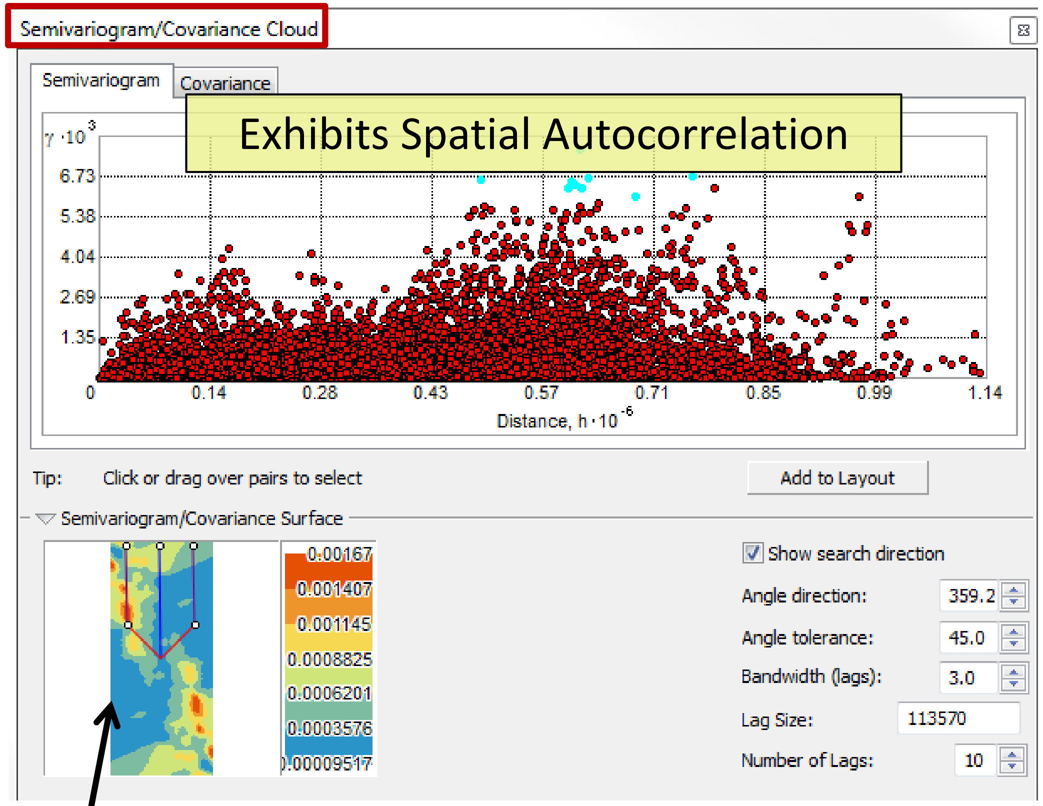


Strong ~NE-SW "U-shaped" Trend

Weaker ~NW-SE linear Trend

Data Exploration:
Spatial Autocorrelation & Directional Influences

- Variogram Analysis:
  - Look for correlation with distance
  - Look for directional trends among pairs of points
    - ❑Semivariogram/ Covariance Cloud tool



Directional Influence shown here – pairs in NNW-SSE direction show largest covariance over shortest distances

ArcGISKriging Processing Steps

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Mapping Ozone Concentration

1. Incorporate results of Data Exploration into Model selection
   - This example:
     - remove underlying trends discovered during data exploration *that have a rational explanation.* (Analysis is then performed on residuals and trend surface is added back into final surface) = *"Detrending"*
     - Remove directional trends between pairs of points – in certain directions closer points are more alike than in other directions = *"anisotropy removal"*
     - *Lag Size – Important! Rule of Thumb: (Lag size) x (# of lags) = 0.5 largest distance (range) among points*

Mapping Ozone Concentration – Interpolation & Cross Validation

2. Define search neighborhood for interpolation (c.f. I.D.W.)
   - Use a search ellipse (or circle) to find nearest neighbors; specify radii of ellipse, min. & max. number of points per sectors

3. Examine Cross Validation plot
   - Predicted vs. Measured for subset(s) of the data
     - "Mean error" should be close to zero
     - "RMS error" and "mean standardized error" should be small
     - "RMS standardized error" should be close to one.

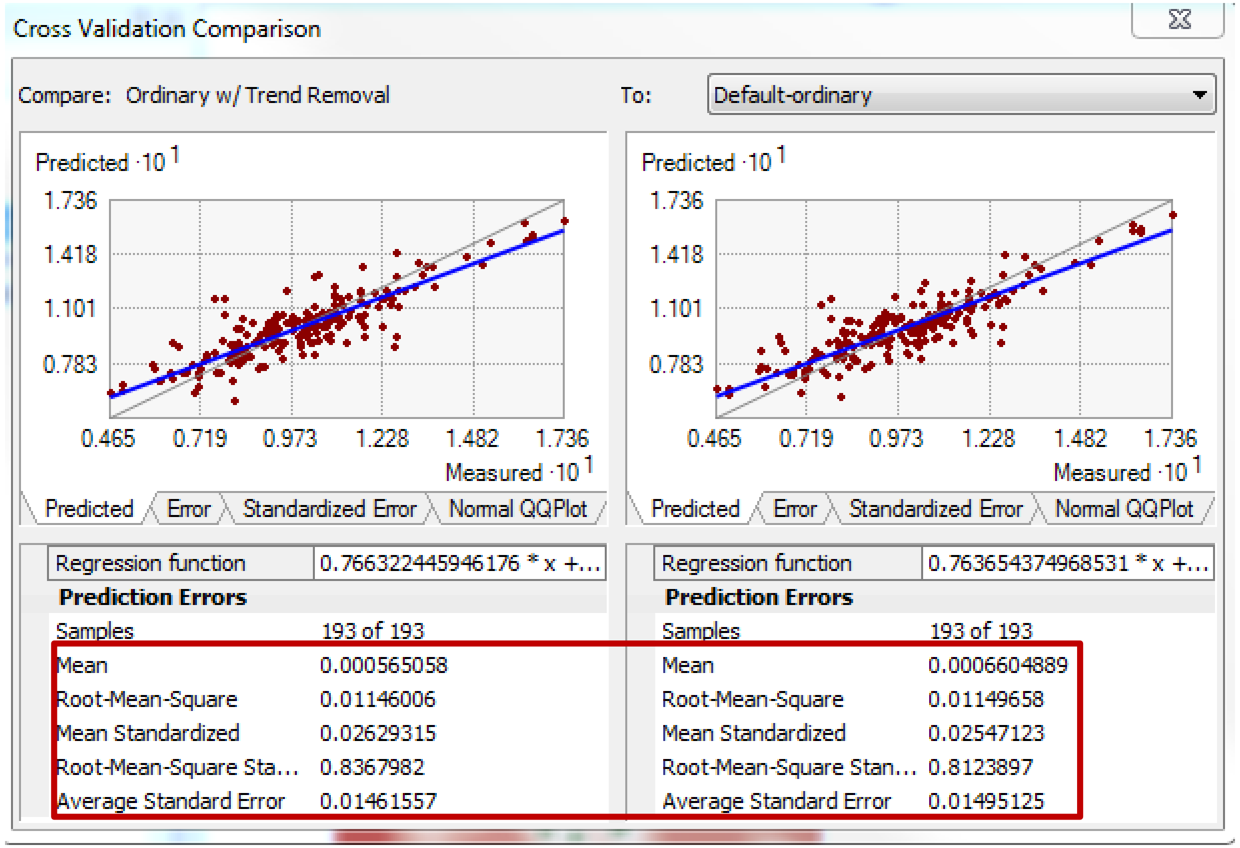
ArcGIS Kriging Processing Steps

1. Add and display the data
2. Explore the data's statistical properties
3. Select a model to create a surface – make a prediction map!
4. Assess the result – Cross Validation Plots
5. Compare to other models

Comparing Model Results

Cross validation comparisons:

- "Mean error" should be close to zero
- "RMS error" and "mean standardized error" should be small
- "RMS standardized error" should be close to one.



Probability Mapping with Indicator Kriging

- Task: Make a map that show the probability of exceeding a critical threshold, e.g. 0.12 ppm ozone for an 8 hr. period
- Technique:
  - Transform data to a series of 0s and 1s according to whether they are above or below the threshold
  - Use a semivariogram on transformed data; interpret indicator prediction values as probabilities