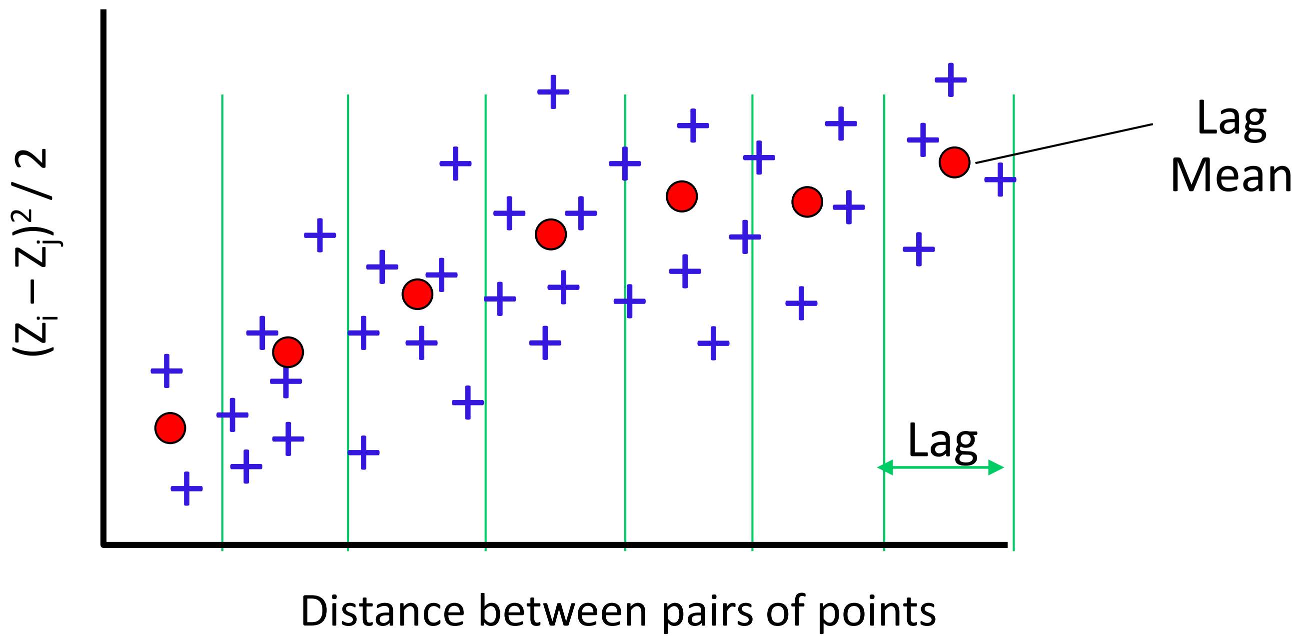
Spatial Interpolation & Geostatistics



Spatial Interpolation

- Determination of unknown values or attributes on the basis of values nearby
  - Used for data that define continuous fields
    - E.g. temperature, rainfall, elevation, concentrations
    - Contouring, raster resampling are applications already discussed

Spatial Interpolation = Spatial Prediction

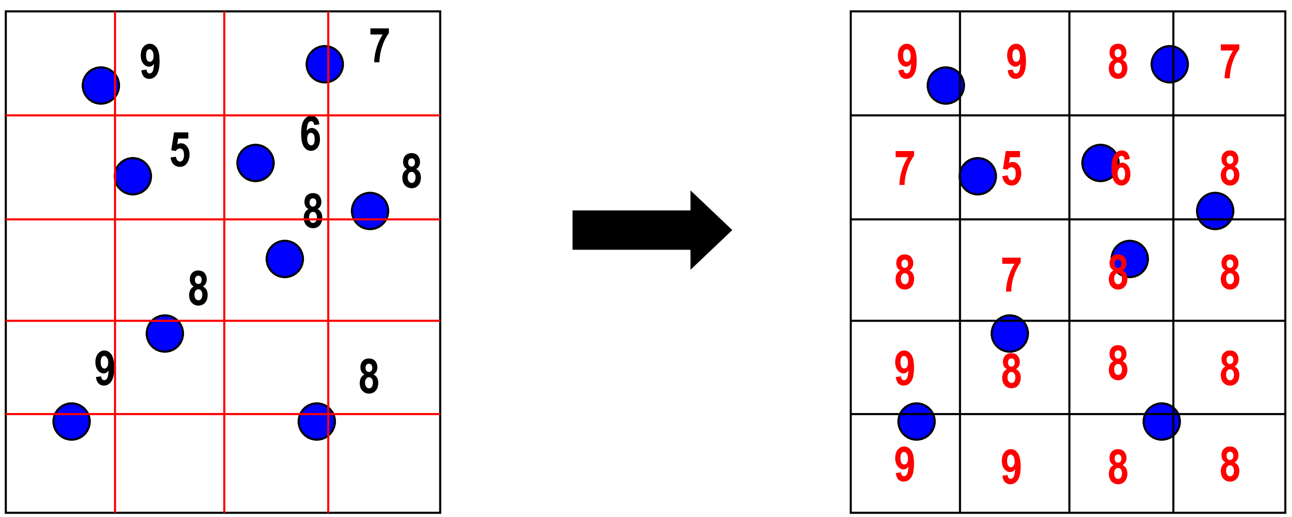
- Equivalent of contouring point measurements by hand, but with verifiable, quantitative methods. HOW?

Root of All Interpolation Methods in Tobler's Law

- "All places are related, but nearby places are related more than distant places"
  - Corollary: fields vary smoothly, slowly and show strong "spatial autocorrelation" – attribute(s) and location are strongly correlated: $Z_i = f\,(x_i, y_i)$

Spatial Interpolation to a Grid of Values

- Interpolate between variably spaced data to a uniform grid of values, a surface raster



Interpolation Methods – Many!

- All address the meaning of "near" in Tobler's law differently
  - How does space make a difference?
  - Statistical mean not best predictor if Tobler's law is true

Interpolation methods - IDW

- Inverse Distance Weighting (IDW)
  - Assumes influence of adjacent points decreases with distance

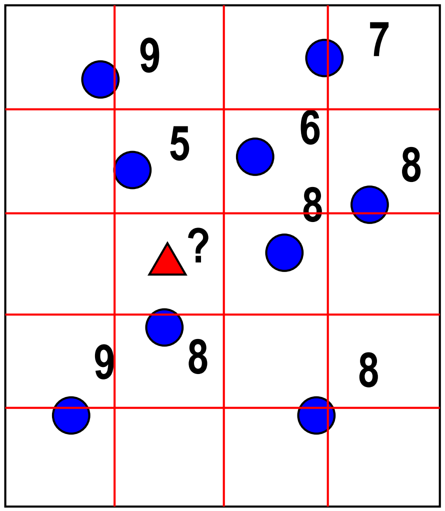
$$Z_0 = \frac{\sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i}$$

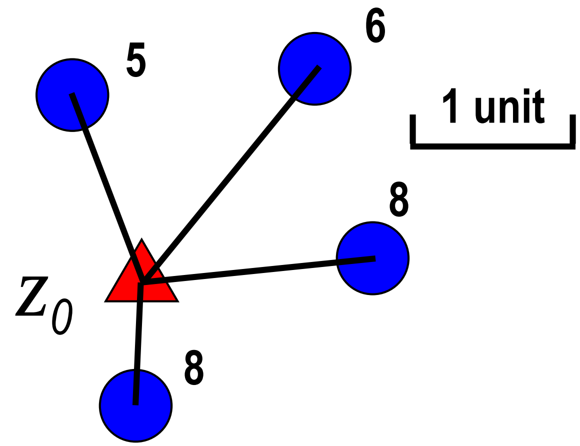
Where: Z_0 = value of estimation point

Z_i = value of neighboring point

W_i = weighting factor; e.g. = 1/(distance from neighbor)^2

Inverse Distance Weighting





On basis of four nearest neighbors:

$$z_0 = (8/(1)^2 + 8/(2)^2 + 6/(2.5)^2 + 5/(2)^2)/(1.66)$$

$$z_0 = (8.0 + 2.0 + 0.96 + 1.25)/(1.66) = 7.36$$

Inverse Distance Weighting (I.D.W.)

- Unknown value is the average of the observed values, weighted by inverse of distance, squared
  - Distance to point doubles, weight decreases by factor of 4

- Can alter IDW by:
  - Alter number of closest points
  - Choose points by distance/search radius
  - Weight be directional sectors
  - Alter distance weighting; e.g. cube instead of square

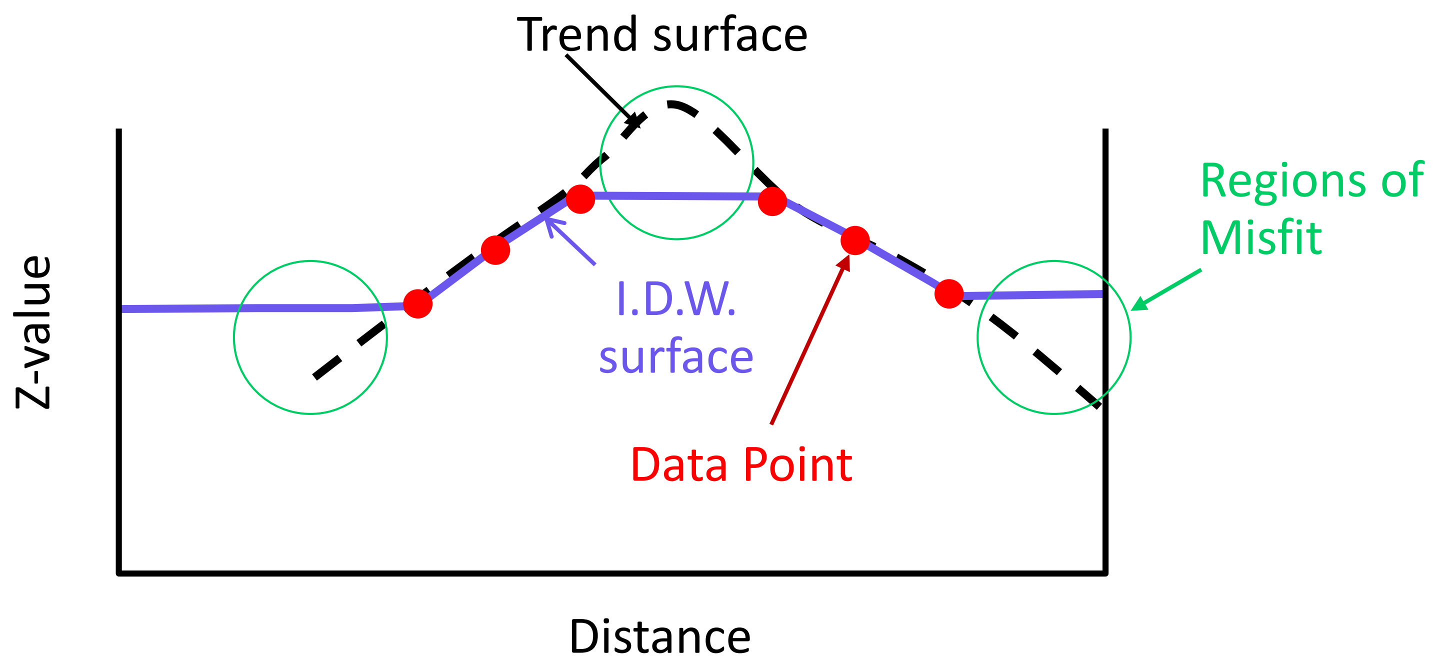
I.D.W. Characteristics

- Is an *Exact Method* of interpolation – will return a *measured value* when applied to measured point.
  - Will not generate smoothness or account for trends, unlike methods that are “*inexact*”
  - Answer (a surface) passes through data points

- Weights never negative –> interpolated values can never be less than smallest z or greater than largest z. “Peaks” and “pits” will never be represented.

I.D.W. Characteristics

- No “peaks” or “pits” possible; interpolated values must lie within range of known values



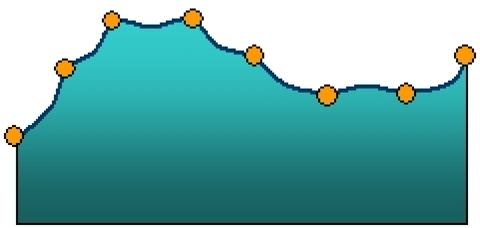
Interpolation Methods - Others

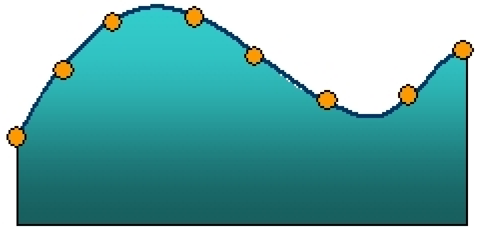
- IDW is inappropriate for values that don't decrease as a function of distance (e.g. topography)
- Other *deterministic*, exact methods:
  - Spline
  - Natural Neighbor

Exact Methods - Spline

- Fit *minimum curvature* surface through observation points; interpolate value from surface
- Good for gently varying surfaces
  - E.g. topography, water table heights
- Not good for fitting large changes over short distances
- Surface is allowed to exceed highest and be less than lowest measured values

Exact Methods: IDW vs. Spline





(images from ArcGIS 9.2 Help files)

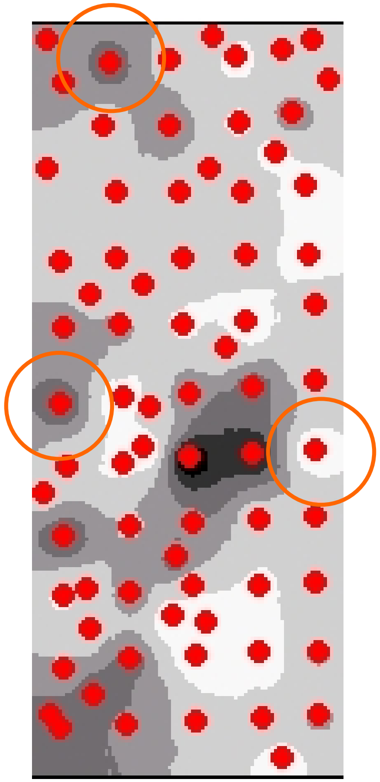
IDW:

- No predicted highs or lows above max. or min. values
- No smoothing; surface can be rough

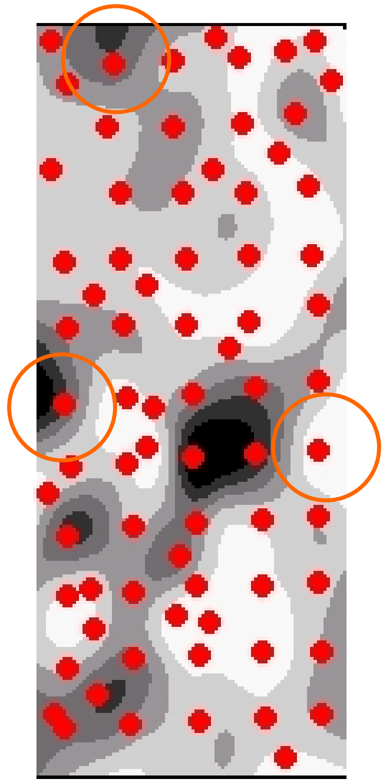
Spline:

- Minimum curvature result good for producing smooth surfaces
- Can't predict large changes over short distances

Comparisons-I.D.W. vs. Spline



IDW, 6 nearest, contoured for 6 classes



Spline, contoured for same 6 classes

- Note smoothing of Spline – less "spikey"
- IDW contours less continuous, fewer inferred maxima and minima

Inexact (Approximate) Methods

Inexact = Answer (surface) need not pass through input data points

- Trend surface –curve fitting by least squares regression
  - Deterministic – one output, no randomness allowed
- Kriging – weight by distance, consider trends in data
  - Stochastic – incorporates randomness

Approximate Methods - Trend

- Fits a polynomial to input points using least squares regression.
- Resulting surfaces minimize variance w.r.t. input values, i.e. sum of difference between actual and estimated values for all inputs is minimized.
- Surface rarely goes through actual points
- Surface may be based on all data ("Global" fit) or small neighborhoods of data ("Local" fits).

Trend Surfaces

Equations are either:

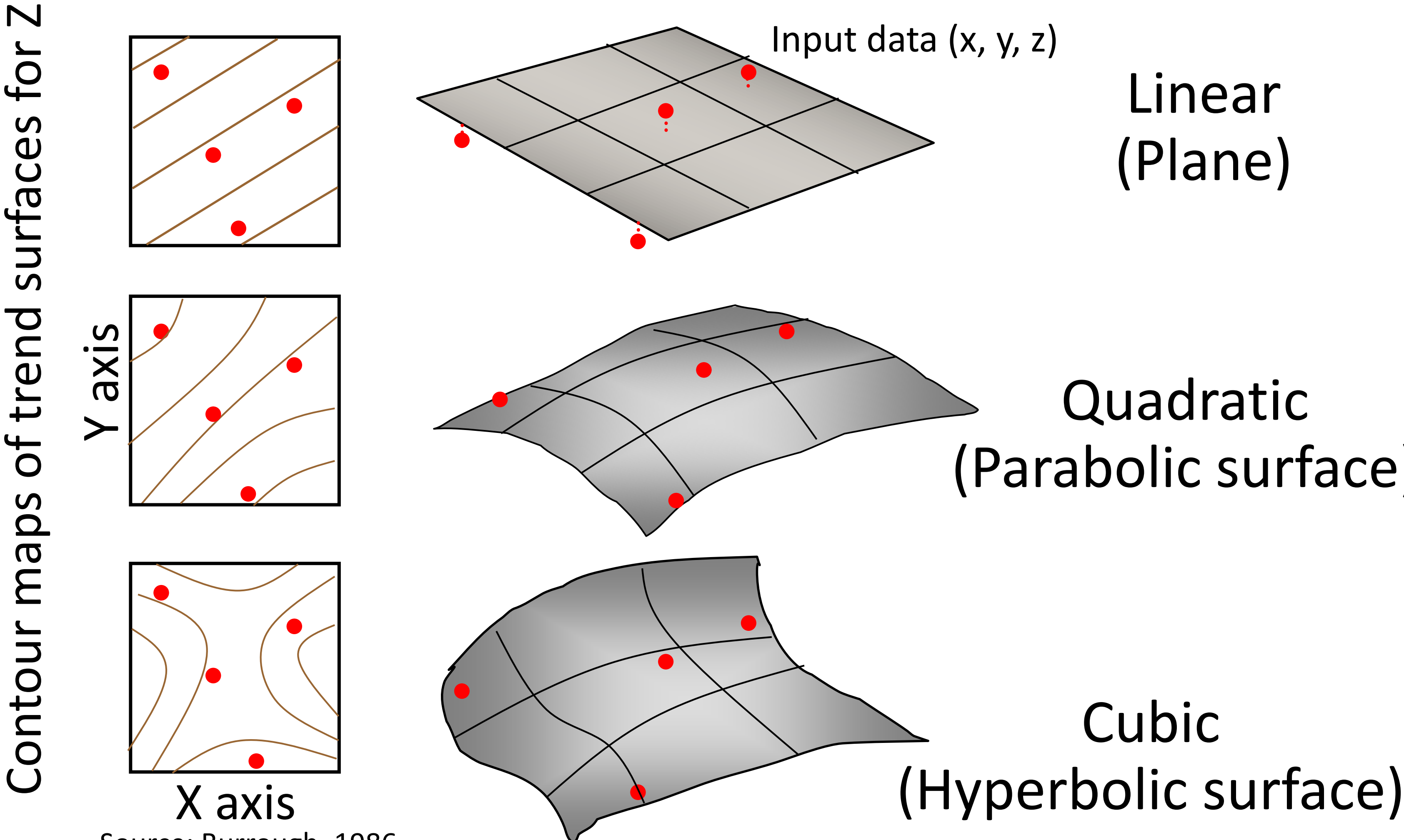
- Linear – *1st Order: fit a plane*
  - $Z = a + bX + cY$
- Quadratic – *2nd Order: fit a plane with one bend (parabolic)*
  - $Z = (1^{st}\ \text{Order}) + dX^2 + eXY + fY^2$
- Cubic – *3rd Order: fit a plane with 2 bends (hyperbolic)*
  - $Z = (2^{nd}\ \text{Order}) + gX^3 + hX^2Y + iXY^2 + Y^3$

  Where:

  a, b, c, d, etc. = constants derived from solution of simultaneous equations

  X, Y = geographic coordinates

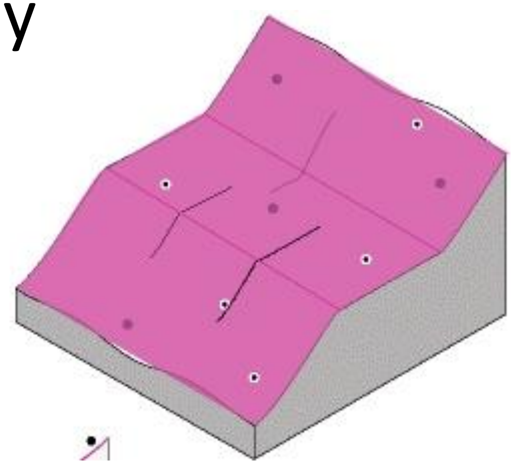
Trend Surfaces – "Global Fitting"

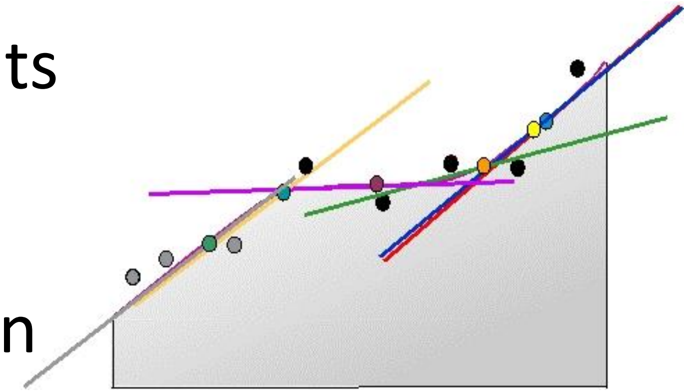


Source: Burrough, 1986

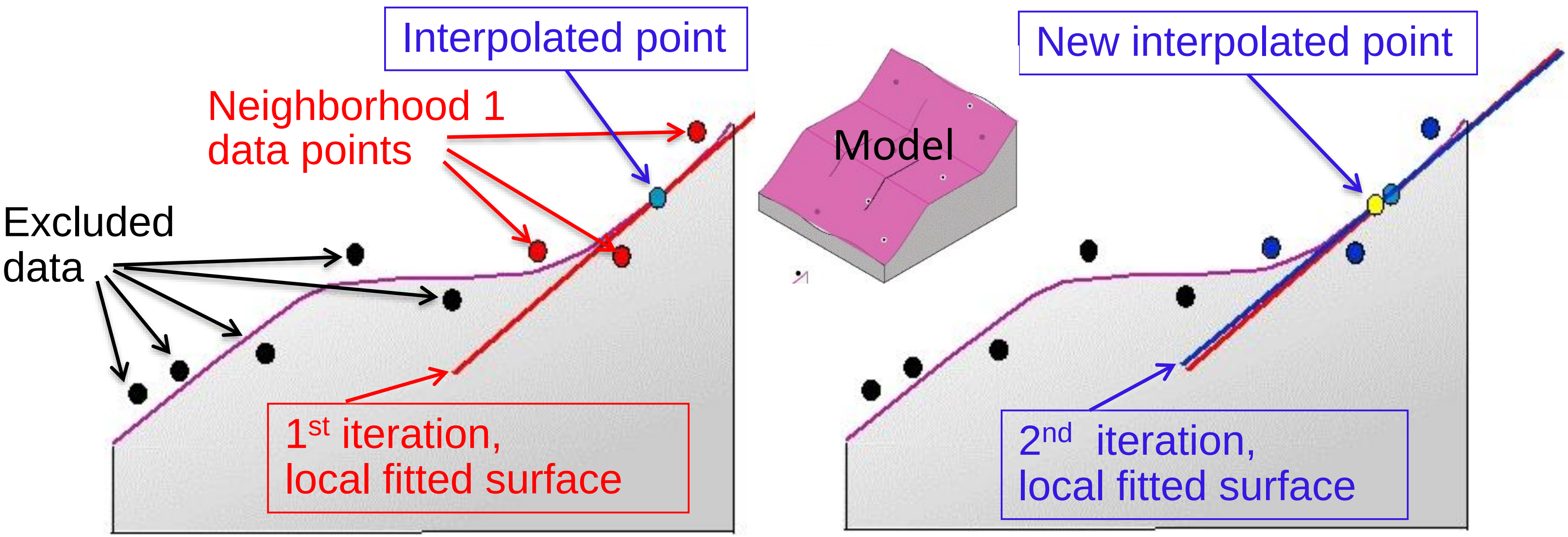
Trend Surfaces – Local fitting

- Local Polynomial Interpolation fits many polynomials, each within specified, overlapping "neighborhoods".
- Neighborhood surface fitting is iterative; final solution is based on minimizing RMS error
- Final surface is composed of best fits to all neighborhoods
- Can be accomplished with tool in ESRI Geostatistical Analyst extension





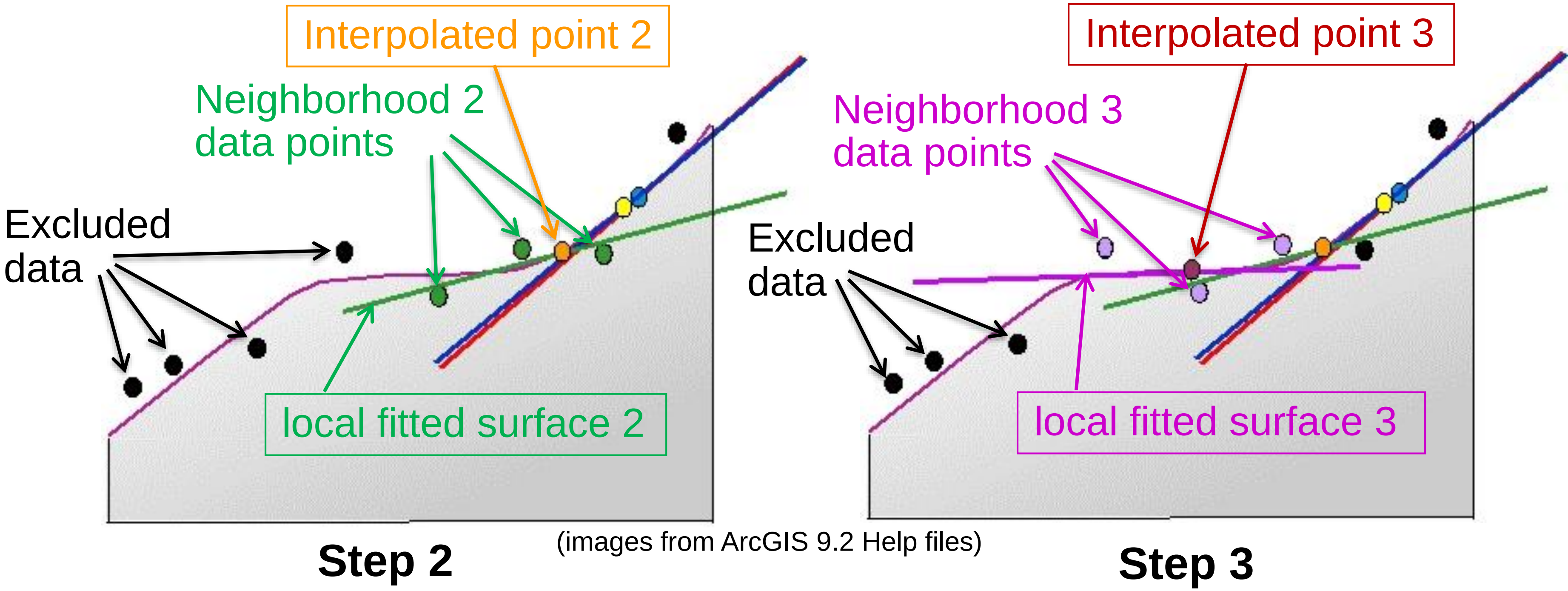
Trend Surfaces – Local fitting



Step 1 (images from ArcGIS 9.2 Help files) **Step 1; 2nd Iteration**

- 2-D profile view of a model surface
  - Neighborhood 1 points (red) are being fit to a plane by iteration (2 steps are shown) and an interpolated point is being created

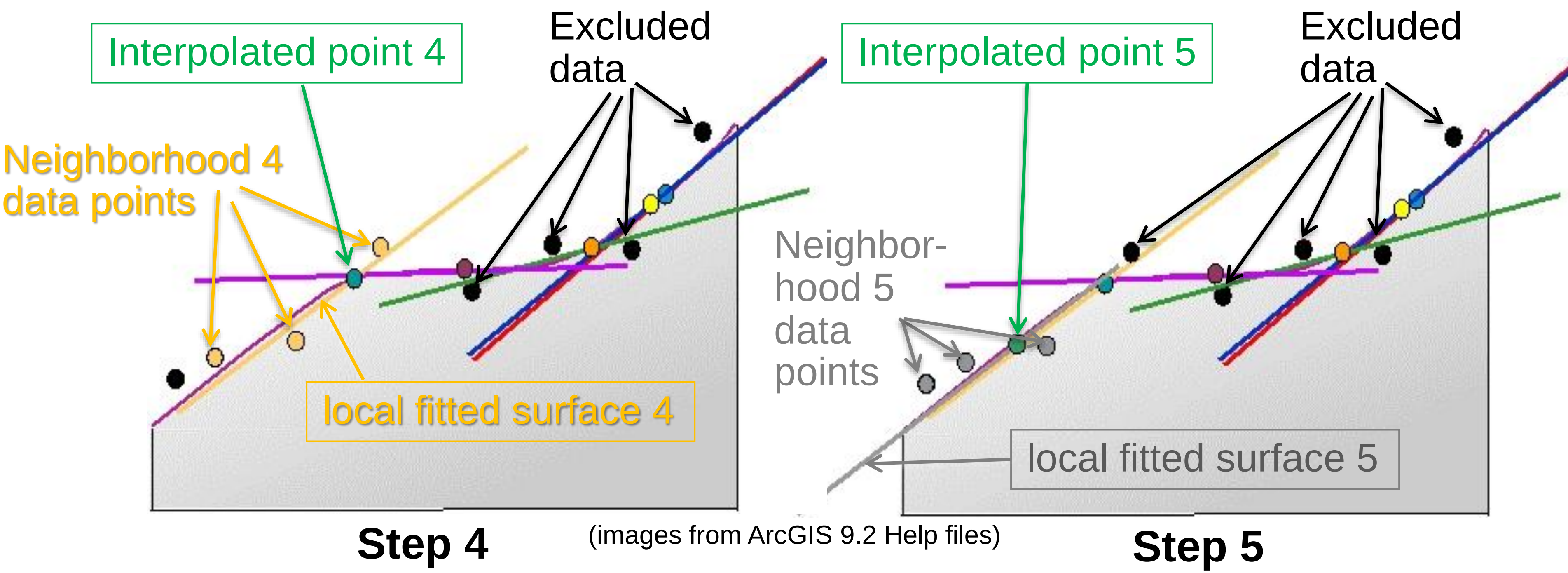
Trend Surfaces – Local fitting, Step 2



(images from ArcGIS 9.2 Help files)

- Model surface generated by many local fits
  - Note that several neighborhoods share some of the same data points: neighborhoods overlap

Trend Surfaces – Local fitting, Step 3

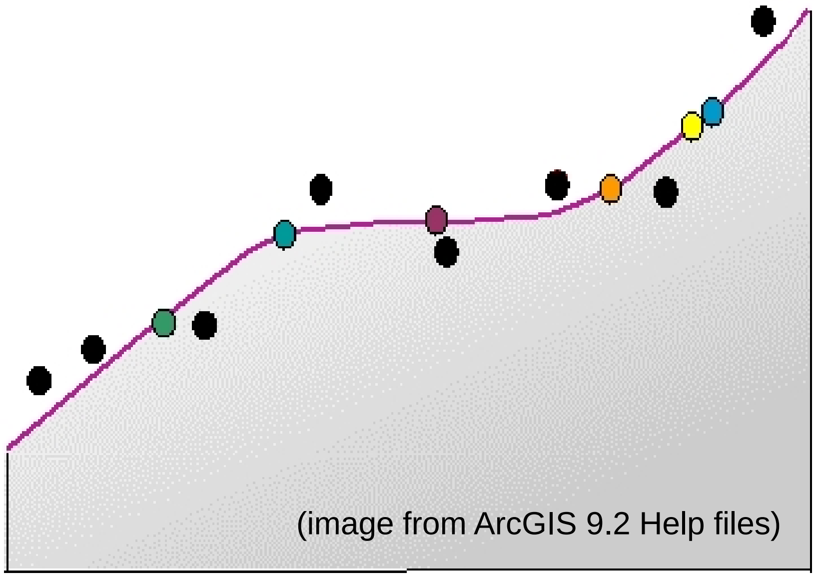


(images from ArcGIS 9.2 Help files)

- Five different polynomials generate five local fits; in this example all are 1st Order.

Trend Surfaces – Local fitting, Step 4

- Result:



Original data points are black

Interpolated points are in colors

- Note that model surface (purple) passes through interpolated points, not measured data points.

Why Trend, Spline or IDW Surfaces?

- No strong reason to assume that z correlated with x, y in these simple ways
- Fitted surface doesn't pass through all points in Trend
- *Data aren't used to help select model*
- → Exploratory, *deterministic* techniques, but theoretically weak

Deterministic vs. Geostatistical Models

- Deterministic: purely a function of distance
  - No associated uncertainties are used or derived
  - E.g. IDW, Trend, Spline

- Geostatistical: based on statistical properties
  - Uncertainties incorporated and provided as a result
  - **Kriging**

Approximate Methods - Kriging

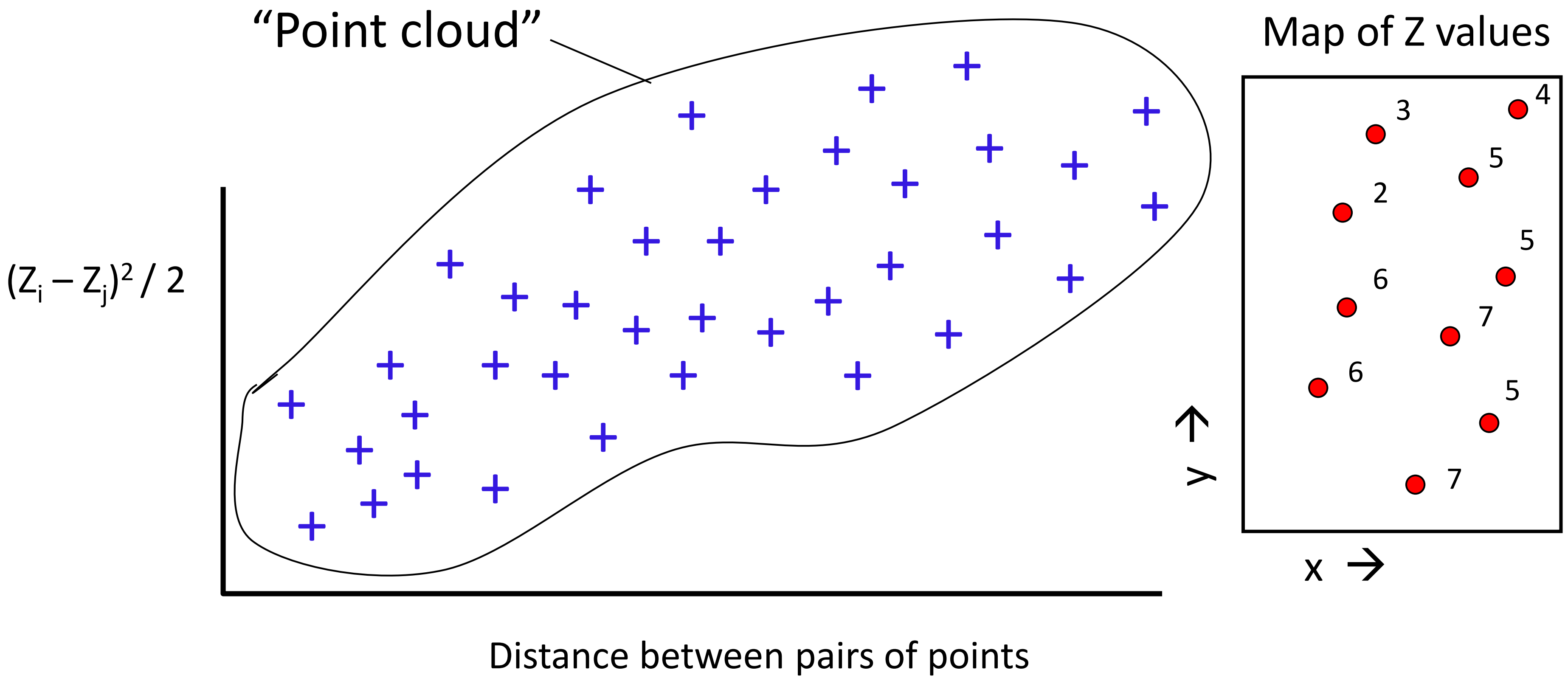
- Kriging
  - Another inverse distance method
  - Considers distance, cluster and spatial covariance (autocorrelation) – look for patterns in data
  - Fit function to selected points; look at correlation, covariance and/or other statistical parameters to arrive at weights – interactive process
  - Good for data that are spatially or directionally correlated (e.g. element concentrations)

Kriging

- Look for patterns over distances, then apply weights accordingly.
- Steps:
  1) Make a description of the spatial variation of the data - *variogram*
  2) Summarize variation by a function
  3) Use this model to determine interpolation weights

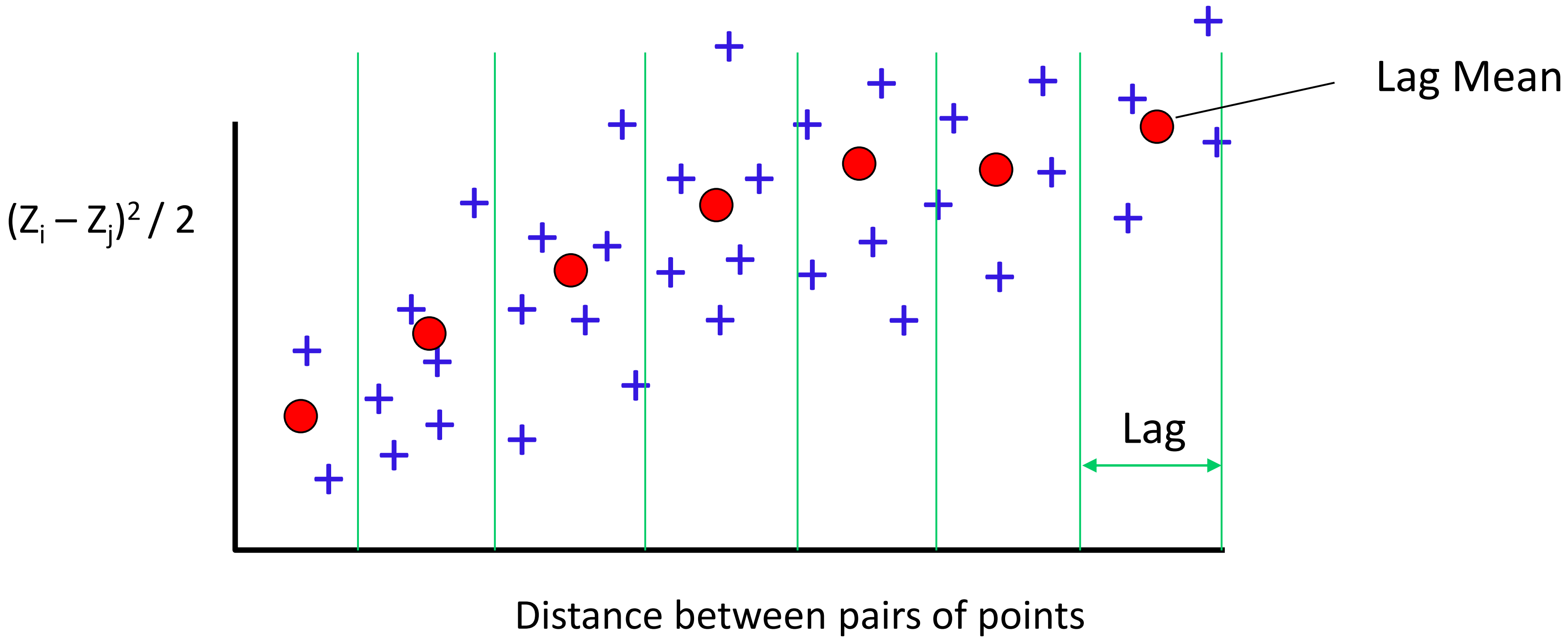
Kriging – Step 1

- Describe spatial variation with *Semivariogram*



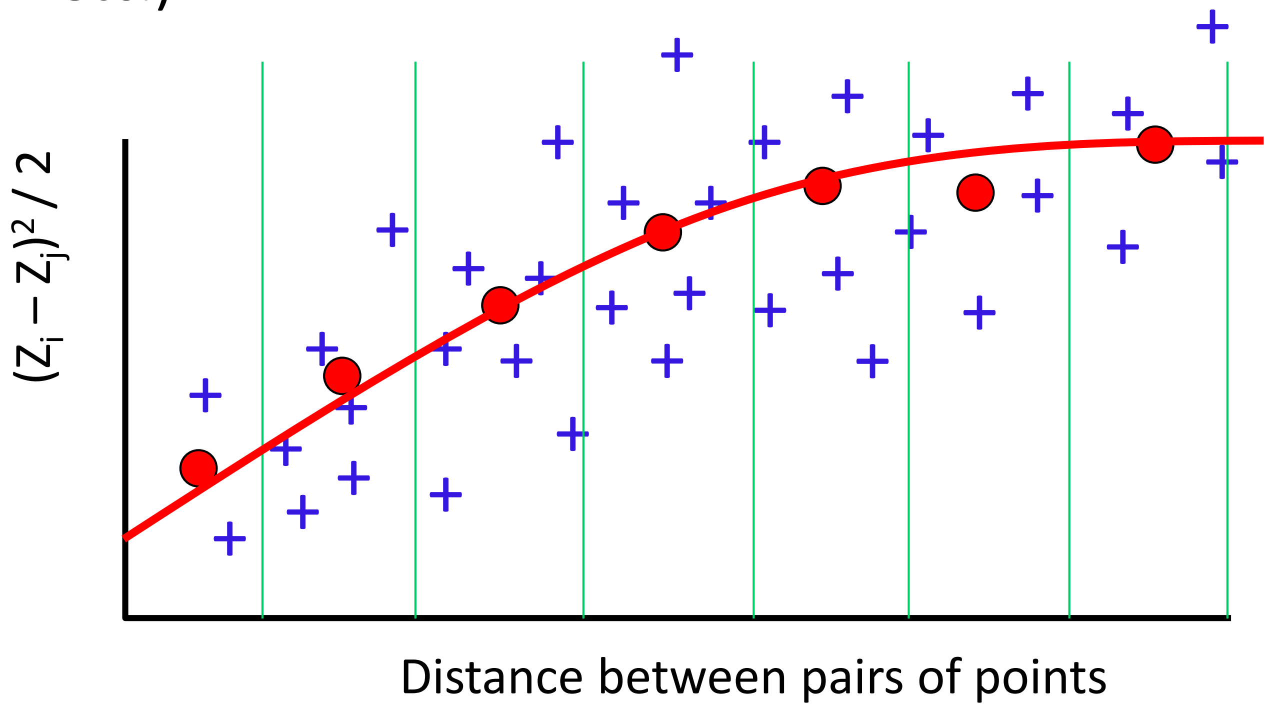
Kriging – Step 1

- Divide range into series of "lags" ("buckets", "bins")
- Find mean values of lags



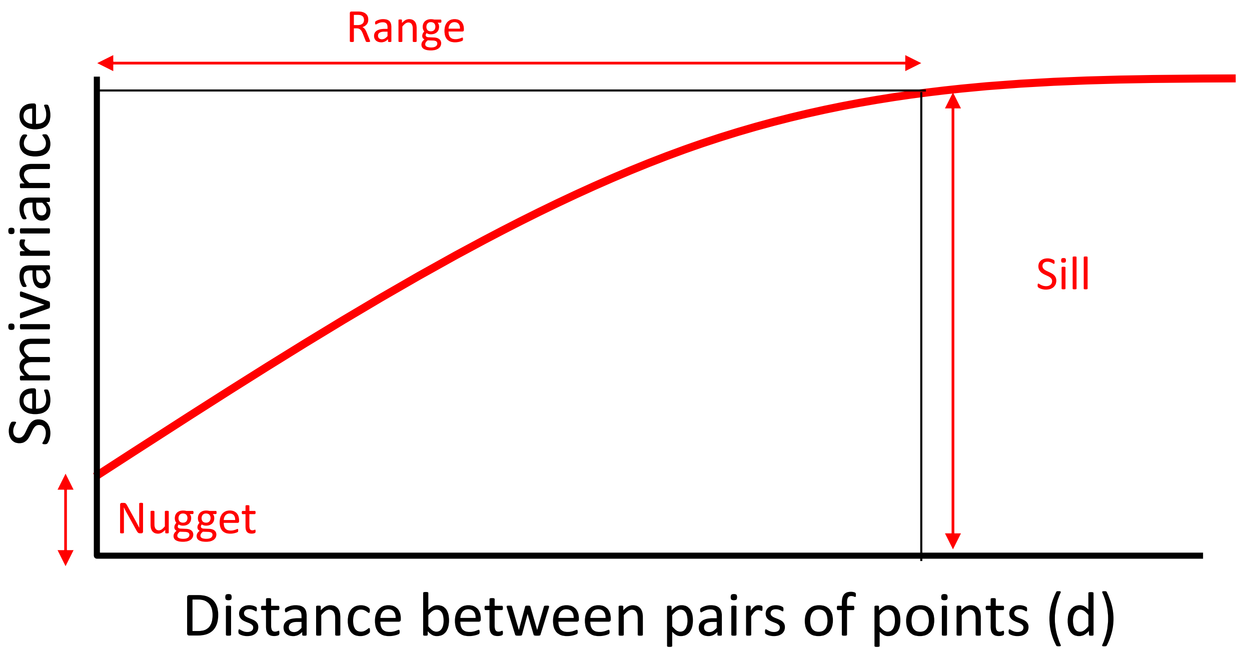
Kriging – Step 2

- Summarize spatial variation with a function
  - Several choices possible; curve fitting defines different types of Kriging (circular, spherical, exponential, gaussian, etc.)



Kriging – Step 2

- Key features of fitted variogram:



Nugget: semivariance at d = 0

Range: d at which semivariance is constant

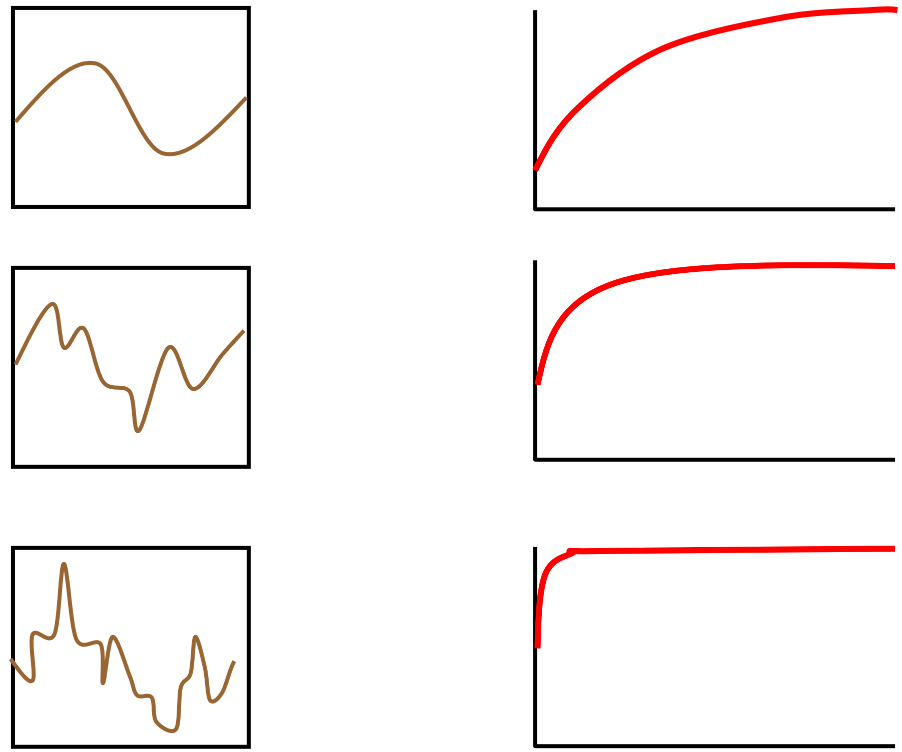
Sill: constant semivariance beyond the range

Kriging – Step 2

- Key features of fitted variogram:
  - Nugget – Measure of uncertainty of z values; precision of measurements
  - Range – No structure to data beyond the range; no correlation between distance and z beyond this value
  - Sill – Measure of the approximate total variance of z

Kriging – Step 2

- Model surface profiles and their variograms:
  - As local variation in surface increases, *range decreases, nugget increases*



Source: O'Sullivan and Unwin, 2003

Kriging – Step 3

- Determine Interpolated weights
  - Use fitted curve to arrive at weights – not explained here; see O'Sullivan and Unwin, 2003 for explanation
  - In general, nearby values are given greater weight (like IDW), but direction can be important (e.g. "shielding" can be considered)

Review:

Deterministic vs. Geostatistical Models

- *Deterministic*: interpolation purely a function of distance
  - No associated uncertainties are used or derived
  - E.g. IDW, Trend, Spline

- *Geostatistical*: interpolation is statistically based
  - Uncertainties incorporated and provided as a result
    - **Kriging** – *next time*