Technical Note of A 10-Layer Soil Moisture and Temperature Model

(DRAFT)

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1. Soil Moisture

1.2 Introduction

The description of this section and the related code (**WATER**) are modified from LSM code of G. Bonan version 1 in CCM3 (Bonan, 1996). Robert Dickinson wrote the first version of the code in July 1996.The primary differences from LSM are (1) vapor pressure boundary condition, (2) clean-up of no longer needed cures of previous instabilities, (3) extension of layer structure to 10 layers, with the layer structure defined as in CCM by definition of layer levels and interfaces taken as half way between levels which give second order accuracy for the flux calculations with uneven layers versus first for more standard differencing,(4) exponential decrease of k_s (saturation hydraulic conductivity) added, (5) a water table level determination level added including highland and lowland levels and fractional area of wetland (water table above the surface). Runoff is parameterized from the lowlands in terms of precipitation incident on wet areas and a base flow, where these are estimated using ideas from TOPMODEL (Beven and Kirkby, 1979; Famiglietti and Wood, 1994; Stieglitz et al., 1997).

Z.-L. Yang checked code and interfaced to frozen soil and snow in October 96 which are currently the same as in Dickinson et al. (1993). The performance of the snow module in the early BATS version has been documented in Yang et al. (1997).

Additional improvements would be using DEMS to better estimate distribution of water tables (project being done by Famiglietti and student) and by improving the ET calculations by use of separate soil water distributions for the highlands and lowlands (simply by prescribing them relative to mean profile, or perhaps better by doing separate soil water calculations for each separate water table region, with some TOPMODEL related expressions for fluxes between regions - perhaps best to express in terms of transfers between the lowest model layers, using vertical diffusion and the $s > 1$adjustment algorithm to redistribute between other layers.

1.2 Layer Structure

Length units in **WATER** are all mm; **TEMPERATURE** subroutine uses the same soil layer structure but lengths are in m.

The depth of soil layer *i* at the full level (Fig. 1), or node depth, z_i, in mm, is

$$z_i = 25(\exp(0.5(i - 0.5)) - 1) \tag{1.1}$$

and the thickness of each layer is

$$\Delta z_i = \begin{cases} 0.5(z_1 + z_2) & i = 1 \\ 0.5(z_{i+1} - z_{i-1}) & i = 2,3,\ldots, N-1 \\ (z_N - z_{N-1)} & i = N \end{cases} \tag{1.2}$$

The depth at the half level or interface, $z_{h,i}$, in mm, is given as

$$z_{h,i} = \begin{cases} 0.5(z_i + z_{i+1}) & i = 1,2,\ldots, N-1 \\ z_N + 0.5\Delta z_N & i = N \end{cases} \tag{1.3}$$

Note that $z_0 = z_{h,0} = 0$. The layer thickness defined above is the thickness between two interfaces except $i = N$. $\Delta z_1 = z_{h,1}$. The exponential form of (1.1) is to obtain more soil layers near the soil surfacewhere the soil water gradient is generally strong (Deardorff, 1978; Dickinson, 1984).

One of the assumptions in the TOPMODEL is the exponential decay of saturated hydraulic conductivity with the depth. This can be written as

$$k_{s,i} = k_s \exp(-z_{h,i} / h_k) \tag{1.4}$$

where k_s is the saturated hydraulic conductivity at the soil surface, $k_{s,i}$ is the saturated hydraulic conductivity at the interface depth $z_{h,i}$, and h_k is a length scale which can be determined by soil type and topography (Famiglietti and Wood, 1994).

The fraction of roots ($f_{root,i}$) located between pairs of interfaces is specified as

$$f_{root,i} = \exp(-z_{h,i-1} / h_{root}) - \exp(-z_{h,i} / h_{root}) \quad (1.5)$$

where h_{root} is the root distribution scale. If we take h_{root} =500mm, h_k =500mm, and $k_s = 6.3\times10^{-3}$ mms^{-1}, we obtain the values for z_i, $\Delta z_i, z_{h,i}, k_{s,i}$ and $f_{root,i}$ for the 10 soil layers as given in Table 1.

Table 1. The Model Layer Structure and Parameters

Index (i)	z_i (mm)	Δz_i (mm)	$z_{h,i}$ (mm)	$k_{s,i}$ (mms^{-1})	$f_{root,i}$
1	7.1	17.5	17.5	6.08×10^{-3}	0.0344
2	27.9	27.6	45.1	5.76×10^{-3}	0.0518
3	62.3	45.5	90.6	5.26×10^{-3}	0.0794
4	119.0	75.0	166.0	4.52×10^{-3}	0.1160
5	212.0	124.0	289.0	3.53×10^{-3}	0.1570
6	366.0	204.0	493.0	2.35×10^{-3}	0.1880
7	620.0	336.0	829.0	1.20×10^{-3}	0.1830
8	1040.0	554.0	1380.0	3.96×10^{-4}	0.1280
9	1730.0	913.0	2300.0	6.38×10^{-5}	0.0528
10	2860.0	1140.0	3430.0	6.57×10^{-6}	0.0091

1.3 Soil Moisture Computations

Soil water conservation in the vertical dimension requires

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} \quad (1.6)$$

where θ is the volumetric soil water content ($mm^3 mm^{-3}$), t is time (s), z is height (mm) above soil surface, positive upward. q is the soil water flux (mms^{-1}), positive upward, and can be described by Darcy's law

$$q = -k\left[\frac{\partial(\Psi + z)}{\partial z}\right] \quad (1.7)$$

$$= -k\left(\frac{\partial \Psi}{\partial z} + 1\right)$$

$$= -k\left(\frac{\partial \Psi}{\partial \theta}\frac{\partial \theta}{\partial z} + 1\right)$$

where k is the hydraulic conductivity (mms^{-1}), and Ψ is the soil matric potential (taking negative values, opposite to soil suction) (mm). Note that there are many choices of signs used to denote directions of z and q in the soil science. The rule is that when z and q are defined in the same direction, the first sign on the right hand side (RHS) of both (1.6) and (1.7) takes -; when z and q are defined in the opposite direction, the first sign in the RHS of both (1.6) and (1.7) takes +. (The same rule also applies when q is replaced by soil heat flux). If z is defined positive upward above the soil surface, the total potential head $h_T = \Psi + z$; if z is defined positive downward below the soil surface, the total potential head $h_T = \Psi + z$.

Substituting (1.7) into (1.6) results in the Richard equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[k\left(\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} + 1\right)\right] \tag{1.8}$$

Equation (1.6) can be solved numerically by dividing the soil domain into m parallel layers in the vertical [see (1.1)-(1.3)] and then integrating downward over each of the model layers (Mahrt and Pan, 1984; Boone and Wetzel, 1996):

$$\int_{-z_{h,i}}^{-z_{h,i-1}} \frac{\partial \theta}{\partial t} dz = -\int_{-z_{h,i}}^{-z_{h,i-1}} \frac{\partial q}{\partial z} dz - \int_{-z_{h,i}}^{-z_{h,i-1}} s dz \tag{1.8a}$$

where $i = 1,2,..., N$, increasing downward (Fig. 1), and s is a soil moisture sink term. The average volumetric water content is defined as

$$\theta_i = \frac{1}{\Delta z_i}\int_{z_{h,i}}^{-z_{h,i-1}} \theta dz \tag{1.8b}$$

where $\Delta z_i = z_{h,i} - z_{h,i-1}$, which is identical to (1.2), θ_i is the average volumetric water content within a layer i. The layer-averaged quantity is defined as being at the center of the layer.

Equation (1.8b) can be written as

$$\Delta z_i \frac{\partial \theta_i}{\partial t} = -(q|_{z=-z_{h,i-1}} - q|_{z=-z_{h,i}}) - s_i \tag{1.8c}$$

s_i is the layer-averaged moisture sink term (e.g., evapotranspiration loss) (mms^{-1}) from layer i, defined in the same manner as (1.8b). Let

$$q_{i-1} = q_{z=-z_{h,i-1}}$$

and

$$q_i = q_{z=-z_{h,i}}$$

Inserting the above two equations into (1.8c) and taking finite difference with time, we have

$$\frac{\Delta z_i \Delta\theta_i}{\Delta t} = q_i^{n+1} - q_{i-1}^{n+1} - s_i \tag{1.8d}$$

$\Delta\theta_i$ is change of θ_i at soil layer i in one time step Δt, The superscript on the RHS of (1.8) means $(n+1)^{th}$ time step, which ensures an implicit integration. q_i is the soil water flux across an interface i, and q_{i-1} is the soil water flux across an interface $i-1$. Hence, the index i refers to the center of the layer (i.e., the full level or node point) for the layer-averaged variables, and to the lower boundary of a layer for the flux terms. Following (1.7), q_i is

$$q_i = -k_i \left[\frac{\partial(\psi + z)}{\partial z} \right]_i \tag{1.9}$$

$$= -k_i \left[\frac{(\psi_i + z|_{z=-z_{i+1}}) - (\psi_{i+1} + z|_{z=-z_{i+1}})}{z|_{z=-z_i} - z|_{z=-z_{i+1}}} \right]$$

$$= -k_i \left[\frac{(\psi_i - z_i) - (\psi_{i+1} - z_{i+1})}{z_{i+1} - z_i} \right]$$

$$= -k_i \left[\frac{(\psi_i - \psi_{i+1}) + (z_{i+1} - z_i)}{z_{i+1} - z_i} \right]$$

where k and ψ, according to Clapp and Hornberger (1978), are,

$$k = k_s \left(\frac{\theta}{\theta_s} \right)^{2B+3} \tag{1.10a}$$

$$\psi = \psi_s \left(\frac{\theta}{\theta_s} \right)^{-B} \tag{1.10b}$$

where k_s is the saturation hydraulic conductivity, ψ_s is the saturation soil matric potential, and B is a constant depending on soil texture. k_i, the hydraulic conductivity across the interface i,
takes the form of

$$k_i = k_{s,i} \left[\frac{0.5(\theta_i + \theta_{i+1})}{\theta_s} \right]^{2B+3} \tag{1.11a}$$

and ψ_i is

$$\psi_i = \psi_s \left(\frac{\theta_i}{\vartheta_s} \right)^{-B} \tag{1.11b}$$

Equations (1.10a) and (1.10b) are widely used in the land-surface models. However, there are other types of formulations which appear in literature and which are widely used in the soil science community (e.g., van Genuchten, 1980). Equations (1.11a) and (1.11b) may have other forms too. Because the flux terms apply at the interface level, their computations require the ``inter-facial" volumetric water content (IFVWC). Boone and Wetzel (1996) have reviewed four commonly used methods to compute the IFVWC. These methods are listed as follows.

(i) IFVWC is computed using the maximum volumetric water content of two neighboring layers (Mahrt and Pan, 1984).

(ii) IFVWC is calculated using the thickness weighted value of the hydraulic conductivities of the surrounding layers (Sellers et al., 1986; Abramopoulos et al., 1988). Abramopoulos et al. have tested linear and logarithmic interpolation of conductivity.

(iii) IFVWC is determined using the arithmetic mean of the volumetric water content of the surrounding model layers (Verseghy, 1991).

(iv) IFVWC is computed using the linear interpolation of the logarithm of the matric potential from the two surrounding layers (Boone and Wetzel, 1996).

Boone and Wetzel (1996) pointed out that Method (i) and Method (ii) could give the nearly identical IFVWC values. They have tested methods (i), (iii) and (iv) in their PLACE land-surface model using both high-resolution (50) and low-resolution (5) vertical grids. They concluded that the simulated soil water contents in the high-resolution grids are insensitive to the three methods tested, but the low-resolution results are very sensitive to the interpolation scheme. In their low-resolution model, Method (iv) provided the results in best agreement with the high-resolution results, while Method (i) gave worst results that the initial wet soil dried out too fast.

However, use of a different method will require a different derivation as seen below. Because ψ and k are non-linear functions of θ, q_i is a function of θ_i and θ_{i+1} following (1.11a) and (1.11b).

At $(n+1)^{th}$ time step, q_i^{n+1} can be approximated as

$$q_i^{n+1} = q_i^n + \frac{\partial q_i}{\partial \theta_i} \Delta\theta_i + \frac{\partial q_i}{\partial \theta_{i+1}} \Delta\theta_{i+1} \tag{1.12a}$$

and similarly,

$$q_{i-1}^{n+1} = q_{i-1}^{n} + \frac{\partial q_{i-1}}{\partial \theta_{i-1}}\Delta\theta_{i-1} + \frac{\partial q_{i-1}}{\partial \theta_{i}}\Delta\theta_{i} \tag{1.12b}$$

Hence q is linearized about $\partial\theta$. Substituting (1.12a) and (1.12b) into (1.8) results in a tridiagonal system of equations for $\partial\theta$, which is,

$$a_i\Delta\theta_{i-1} + b_i\Delta\theta_i + c_i\Delta\theta_{i-1} = r_i, \qquad i = 1,2,..., N \tag{1.13}$$

where

$$a_i = -\frac{\partial q_{i-1}}{\partial \theta_{i-1}} \tag{1.14a}$$

$$b_i = \frac{\partial q_i}{\partial \theta_i} - \frac{\partial q_{i-1}}{\partial \theta_i} - \frac{\Delta z_i}{\Delta t} \tag{1.14b}$$

$$c_i = \frac{\partial q_i}{\partial \theta_{i+1}} \tag{1.14c}$$

$$r_i = s_i + q_{i-1}^n - q_i^n \tag{1.14d}$$

If $2 \le i \le N-1$,

$$q_{i-1}^n = -k_{i-1}\left[\frac{(\psi_{i-1} - \psi_i) + (z_i - z_{i-1})}{z_i - z_{i-1}}\right] \tag{1.15a}$$

$$q_i^n = -k_i\left[\frac{(\psi_i - \psi_{i+1}) + (z_{i+1} - z_i)}{z_{i+1} - z_i}\right] \tag{1.15b}$$

$$\frac{\partial q_{i-1}}{\partial \theta_{i-1}} = -\left[\frac{k_{i-1}}{z_i - z_{i-1}}\frac{\partial \psi_{i-1}}{\partial \theta_{i-1}}\right] - \frac{\partial k_{i-1}}{\partial \theta_{i-1}}\left[\frac{(\psi_{i-1} - \psi_i) + (z_i - z_{i-1})}{z_i - z_{i-1}}\right] \tag{1.15c}$$

$$\frac{\partial q_{i-1}}{\partial \theta_i} = \left[\frac{k_{i-1}}{z_i - z_{i-1}}\frac{\partial \psi_i}{\partial \theta_i}\right] - \frac{\partial k_{i-1}}{\partial \theta_i}\left[\frac{(\psi_{i-1} - \psi_i) + (z_i - z_{i-1})}{z_i - z_{i-1}}\right] \tag{1.15d}$$

$$\frac{\partial q_i}{\partial \theta_i} = -\left[\frac{k_i}{z_{i+1} - z_i}\frac{\partial \psi_i}{\partial \theta_i}\right] - \frac{\partial k_i}{\partial \theta_i}\left[\frac{(\psi_i - \psi_{i+1}) + (z_{i+1} - z_i)}{z_{i+1} - z_i}\right] \tag{1.15e}$$

$$\frac{\partial q_i}{\partial \theta_{i+1}} = \left[\frac{k_i}{z_{i+1} - z_i}\frac{\partial \psi_{i+1}}{\partial \theta_{i+1}}\right] - \frac{\partial k_i}{\partial \theta_{+1i}}\left[\frac{(\psi_i - \psi_{i+1}) + (z_{i+1} - z_i)}{z_{i+1} - z_i}\right] \tag{1.15f}$$

$$\frac{\partial k_{i-1}}{\partial \theta_{i-1}} = \frac{\partial k_{i-1}}{\partial \theta_i} = \frac{k_{i-1}(2B+3)}{\theta_{i-1} + \theta_i} \tag{1.15g}$$

$$\frac{\partial k_i}{\partial \theta_i} = \frac{\partial k_i}{\partial \theta_{i+1}} = \frac{k_i(2B+3)}{\theta_i + \theta_{i+1}} \tag{1.15h}$$

Let

$$n_1 = -(\psi_{i-1} - \psi_i) - (z_i - z_{i-1}) \tag{1.16a}$$

$$n_2 = -(\psi_i - \psi_{i+1}) - (z_{i+1} - z_i) \tag{1.16b}$$

$$d_1 = (z_i - z_{i-1}) / k_{i-1} \tag{1.16c}$$

$$d_2 = (z_{i+1} - z_i) / k_i \tag{1.16d}$$

then, (1.15a)-(1.15f) can be expressed as

$$q_{i-1}^n = n_1 / d_1 \tag{1.17a}$$

$$q_i^n = n_2 / d_2 \tag{1.17b}$$

$$\frac{\partial q_{i-1}}{\partial \theta_{i-1}} = \left[-\frac{\partial \psi_{i-1}}{\partial \theta_{i-1}} + \frac{n_1}{k_{i-1}} \frac{\partial k_{i-1}}{\partial \theta_{i-1}} \right] / d_1 \tag{1.17c}$$

$$\frac{\partial q_{i-1}}{\partial \theta_i} = \left[\frac{\partial \psi_i}{\partial \theta_i} + \frac{n_1}{k_{i-1}} \frac{\partial k_{i-1}}{\partial \theta_i} \right] / d_1 \tag{1.17d}$$

$$\frac{\partial q_i}{\partial \theta_i} = \left[-\frac{\partial \psi_i}{\partial \theta_i} + \frac{n_2}{k_i} \frac{\partial k_i}{\partial \theta_i} \right] / d_2 \tag{1.17e}$$

$$\frac{\partial q_i}{\partial \theta_{i+1}} = \left[\frac{\partial \psi_{i+1}}{\partial \theta_{i+1}} + \frac{n_2}{k_i} \frac{\partial k_i}{\partial \theta_{i+1}} \right] / d_2 \tag{1.17f}$$

If $i = 1$, (1.8) can be written as,

$$\frac{\Delta \theta_i \Delta z_i}{\Delta t} = -q_{\inf il}^{n+1} + q_i^{n+1} - s_i \tag{1.18}$$

where

$$q_{\inf il} = R_s - G_w \tag{1.19}$$

with

$$G_w = P_w - E_w + S_m + D_r / \Delta t \tag{1.20}$$

$$R_s = \max(0, f_c G_w) \tag{1.21}$$

where P_w is rainfall after interception by canopy, E_w is soil surface evaporation, S_m is snowmelt, D_r is drip of rain water from canopy, and Δt is time step.

If $G_w > 0$,

$$\frac{\partial R_s}{\partial \theta_1} = f_c \frac{\partial G_w}{\partial \theta 1} = -f_c \frac{\partial E_w}{\partial \theta_1} = -f_c s_{damp} \tag{1.22}$$

If $G_w \le 0$, $R_s = 0$ and $\frac{\partial R_s}{\partial \theta_1} = 0$.

The surface infiltration, $q_{\inf il}$ at time step $n+1$ may be given as

$$\begin{aligned} q_{\inf il}^{n+1} &= q_{\inf il}^{n} + \frac{\partial q_{\inf il}}{\partial \theta_1} \Delta\theta_1 \\ &= q_{\inf il}^{n} + \left[\frac{\partial R_s}{\partial \theta_1} - \frac{\partial G_w}{\partial \theta_1} \right] \Delta\theta_1 \\ &= q_{\inf il}^{n} + \left[\frac{\partial R_s}{\partial \theta_1} + \frac{\partial E_w}{\partial \theta_1} \right] \Delta\theta_1 \end{aligned} \tag{1.23}$$

Substituting (1.12a) and (1.23) into (1.18) results in a tridiagonal system, with its coefficients equal to

$$a_i = 0 \tag{1.24a}$$

$$b_i = \frac{\partial q_i}{\partial \theta_i} - \left[\frac{\Delta z_i}{\Delta t} + \frac{\partial R_s}{\partial \theta_1} + \frac{\partial E_w}{\partial \theta_1} \right] \tag{1.24b}$$

$$c_i = \frac{\partial q_i}{\partial \theta_{i+1}} \tag{1.24c}$$

$$r_i = s_i + q_{\inf il}^{n} - q_i^{n} \tag{1.24d}$$

where

$$q_i^n = n_2 / d_2 \tag{1.25a}$$

$$\frac{\partial q_i}{\partial \theta_i} = \left[-\frac{\partial \psi_u}{\partial \theta_i} + \frac{n_2}{k_i} \frac{\partial k_i}{\partial \theta_i} \right] / d_2 \tag{1.25b}$$

$$\frac{\partial q_i}{\partial \theta_{i+1}} = \left[\frac{\partial \psi_{i+1}}{\partial \theta_{i+1}} + \frac{n_2}{k_i} \frac{\partial k_i}{\partial \theta_{i+1}} \right] / d_2 \tag{1.25c}$$

If $i = N$, the bottom boundary condition is

$$q_i = -k_N \tag{1.26}$$

Equation (1.8) can be written as,

$$\frac{\Delta\theta_i \Delta z_i}{\Delta t} = -q_{i-1}^{n+1} - k_N^{n+1} - s_i \tag{1.27}$$

where

$$k_N = k_s \left(\frac{\theta_{10}}{\theta_s} \right)^{2B+3} \quad (1.28a)$$

$$k_N^{n+1} = k_N^n + \frac{\partial k_N}{\partial \theta} \Delta\theta_i \quad (1.28b)$$

Substituting above equation and (1.12b) into (1.27), the coefficients for the tridiagonal system are

$$a_i = -\frac{\partial q_{i-1}}{\partial \theta_{i-1}} \quad (1.29a)$$

$$b_i = -(\frac{\partial k_i}{\partial \theta_i} + \frac{\partial q_{i-1}}{\partial \theta_i} + \frac{\Delta z_i}{\Delta t}) \quad (1.29b)$$

$$c_i = 0 \quad (1.29c)$$

$$r_i = s_i + q_{i-1}^n + k_i^n \quad (1.29d)$$

where

$$q_{i-1}^n = n_1 / d_1 \quad (1.30a)$$

$$\frac{\partial k_N}{\partial \theta_i} = \frac{k_N (2B+3)}{\theta_{10}} \quad (1.30b)$$

$$\frac{\partial q_{i-1}}{\partial \theta_{i-1}} = \left[-\frac{\partial \psi_{u-1}}{\partial \theta_{i-1}} + \frac{n_1}{k_{i-1}} \frac{\partial k_{i-1}}{\partial \theta_{i-1}} \right] / d_1 \quad (1.30c)$$

$$\frac{\partial q_{i-1}}{\partial \theta_i} = \left[\frac{\partial \psi_i}{\partial \theta_i} + \frac{n_1}{k_{i-1}} \frac{\partial k_{i-1}}{\partial \theta_i} \right] / d_1 \quad (1.30d)$$

2. Soil Temperature

2.1 Introduction

The description of this section and the related code (**TGROUND**) are modified from LSM code of G. Bonan version 1 in CCM3 (Bonan, 1996). Robert Dickinson wrote the original warm soil code in August 1996. The primary differences from LSM are as follows: (1) use of CCM-like vertical differencing (mesh points specified and interfaces located half way between), and BATS for thermal properties tested against analytic solution, (2) provided heating in phase with latest T_g used and 0.5 hr time steps, reproduced exact solution for diurnal heating to within 2% of peak values and peak values within 1%. Similar errors found for response to semidiurnal forcing; for 4 hr periods errors of 10% of peak noted; to resolve any shorter time scales, both the time step and the 0.025 scaling factor for vertical grid should be reduced. In principle, the top soil layer at best represents the temperature at the first node within the soil which will have a somewhat reduced diurnal amplitude. An accurate surface skin temperature is provided that compensates for this effect and numerical error by tuning the heat capacity of the top layer to give an exact match to analytic solution for diurnal heating.

Cold soil (snow and freezing effects) was added by Z.-L. Yang in October 1996. These include soil freezing blended across ± 0.5 °C degrees as in LSM, and inclusion of in surface thermal properties and temperature calculation. Diurnal variation of surface temperature is an important design criteria that can only be met with several (perhaps 5 or more) layers or use of force restore; for now we have used the latter (cf. Dickinson et al., 1993 for more details). For thin enough snow, we revert to the LSM strategy of simply adding the snow to the top soil layer. This approach kills the diurnal surface temperature variation when the snow depth becomes a significant fraction of the diurnal penetration depth. The force restore treatment introduces an additional prognostic variable T_g for snow/soil surface temperature which reverts to the temperature provided by the first soil layer for thin enough snow. Snow melting is done in either case using the BATS force restore formalism; this allows determination of conductive heat exchange with the underlying surface without many layers-by assuming diurnal oscillation of temperature. Unless otherwise stated, thermal properties use BATS constants and parameterization (Dickinson et al., 1993).

2.2 Soil Temperature Computations

The classical one-dimensional heat diffusion equation is as follows

$$c\frac{\partial T}{\partial t} = -\frac{\partial F}{\partial z} \tag{2.1}$$

where T is the soil temperature (K), c is the volumetric soil heat capacity ($Jm^{-3}K^{-1}$), F is the heat flux, and both z and F are defined positive upward (Fig. 1). F takes the form

$$F = -\lambda\frac{\partial T}{\partial z} \tag{2.2}$$

where λ is the thermal conductivity ($Wm^{-1}K^{-1}$). The energy balance for the i^{th} layer is

$$\frac{c_i \Delta z_i}{\Delta t}(T_i^{n+1} - T_i^{n}) = \alpha(-F_{i-1}^{n} + F_i^{n}) + (1-\alpha)(-F_{i-1}^{N+1} + F_i^{n+1}) \tag{2.3}$$

where T_i is the layer-averaged temperature in layer i, Δz_i is the same as in (1.2) except that its unit is now in m. α is a weight coefficient in the time domain, and its value is between 0 and 1. If $\alpha = 0$, the time differencing is implicit, and if $\alpha = 1$, the time differencing is explicit. If $\alpha = 0.5$, the method is the so-called ``Crank-Nicolson" algorithm. As in the soil water section, the index i refers to the center of the layer (i.e., the full level or node point) for the layer-averaged variables, and to the lower boundary of a layer for the flux terms. Hence, F_i (Wm^{-2}) is the heat flux across an interface i, and F_{i-1} (Wm^{-2}) is the heat flux across an interface $i-1$. Both are computed as follows.

$$F_i = \frac{-\lambda_i (T_i - T_{i+1})}{z|_{z=-z_i} - z|_{z=-z_{i+1}}} = \frac{-\lambda_i (T_i - T_{i+1})}{z_{i+1} - z_i} \tag{2.4a}$$

$$F_{i-1} = \frac{-\lambda_{i-1} (T_{i-1} - T_i)}{z|_{z=-z_{i-1}} - z|_{z=-z_i}} = \frac{-\lambda_{i-1} (T_{i-1} - T_i)}{z_i - z_{i-1}} \tag{2.4b}$$

λ_i is the interface thermal conductivity and is computed using

$$\lambda_i = f_\lambda \left(\frac{\theta_i + \theta_{i+1}}{2} \right) \tag{2.4c}$$

where the functional form f_λ follows that in BATS (Dickinson et al., 1993) as

$$f_\lambda(x) = r_{tex} f_c(x) \left[\frac{2.9 \times 10^{-7} x + 4 \times 10^{-9}}{((1 - 0.6x)x + 0.09)(0.23 + x)} \right] \tag{2.4d}$$

where r_{tex} is the ratio of soil thermal conductivity to that of loam (cf. Dickinson et al., 1993), and $f_c(x)$ is given as

$$f_c(x) = (0.23 + x) 4.186 \times 10^6 \tag{2.4e}$$

In (2.3), c_i is the volumetric soil heat capacity for layer i, and is defined as

$$c_i = f_c(\theta_i) \tag{2.4f}$$

Substituting (2.4a) and (2.4b) into (2.3) results in a tridiagonal system of equations for T, which is,

$$a_i T_{i-1}^{n+1} + b_i T_i^{n+1} + c_i T_{i+1}^{n+1} = r_i, \qquad i = 1,2,..., N \tag{2.5}$$

where if $i = 2,3,..., N-1$,

$$a_i = -(1-\alpha)\frac{\Delta t}{c_i}\frac{\lambda_{i-1}}{\Delta z_i (z_i - z_{i-1})} \tag{2.6a}$$

$$b_i = 1 + (1-\alpha)\frac{\Delta t}{c_i}\left[\frac{\lambda_{i-1}}{\Delta z_i (z_i - z_{i-1})} + \frac{\lambda_i}{\Delta z_i (z_{i+1} - z_i)}\right] \tag{2.6b}$$

$$c_i = -(1-\alpha)\frac{\Delta t}{c_i}\frac{\lambda_i}{\Delta z_i (z_{i+1} - z_i)} \tag{2.6c}$$

$$r_i = T_i^n - \frac{\alpha \Delta t T_i^n}{c_i}\left[\frac{\lambda_{i-1}}{\Delta z_i (z_i - z_{i-1})} + \frac{\lambda_i}{\Delta z_i (z_{i+1} - z_i)}\right] + \frac{\alpha \Delta t}{c_i}\left[\frac{\lambda_{i-1} T_{i-1}}{\Delta z_i (z_i - z_{i-1})} + \frac{\lambda_i T_{i+1}}{\Delta z_i (z_{i+1} - z_i)}\right] \tag{2.6d}$$

If $i = 1$,

$$\frac{c_i \Delta z_i}{\Delta t}(T_i^{n+1} - T_i^n) = h^{n+1} + F_i^{n+1} \tag{2.7}$$

where F_i is defined as in (2.4a), and h is the heat flux into the surface soil layer from the overlying atmosphere, which takes the form

$$h = R_{n,g} - H_{g,a} - LE_{g,a} \tag{2.8}$$

where $R_{n,g}$ is the net radiation at the soil surface (positive downward), and $H_{g,a}$ and $LE_{g,a}$ are, respectively, sensible and latent heat flux from the surface (g) to the overlying atmosphere (a). Because $LE_{g,a}$, $H_{g,a}$ and $R_{n,g}$ are a function of soil surface temperature, h may be represented as follows

$$h^{n+1} = h^n + \frac{\partial h}{\partial T_i}(T_i^{n+1} - T_i^n) \tag{2.9}$$

Substituting (2.4a) and (2.9) into (2.7) gives a tridiagonal system of equations with coefficients equal to

$$a_i = 0 \tag{2.10a}$$

$$b_i = 1 + \frac{\Delta t}{c_i \Delta z_i}\left[\frac{\lambda_i}{z_{i+1} - z_i} - \frac{\partial h}{\partial T_i}\right] \tag{2.10b}$$

$$c_i = -\frac{\Delta t}{c_i \Delta z_i}\frac{\lambda_i}{z_{i+1} - z_i} \tag{2.10c}$$

$$r_i = T_i^n + \frac{h \Delta t}{c_i \Delta z_i} - \frac{T_i^n \Delta t}{c_i \Delta z_i}\frac{\partial h}{\partial T_i} \tag{2.10d}$$

Note that T_1 obtained from above equations is the layer-averaged temperature for $\Delta z_1 = 0.5(z_1 + z_2)$ which has a somewhat reduced diurnal amplitude. In order to obtain temperature at the soil surface (skin temperature), the layer heat capacity can be adjusted by adjusting the layer thickness. Hence, Δz_1 in (2.10b), (2.10c) and (2.10d) can be represented as

$$\Delta z_1 = 0.5(z_1 + c_a z_2) \tag{2.10e}$$

where c_a is a tuned parameter, varying from 0 and 1. Using the algorithm as described in this section, $c_a \approx 0.34$ is found to give best match between T_1 and analytical solution of surface temperature for diurnal heating (see Appendix A and Fig. 2). It should be pointed out that the optimum value of c_a depends on the structure of the soil layers, and value of α. A software is made available to the user who likes to obtain a value of c_a for a different model layer structure or a different value of α. Furthermore, note that the optimum surface temperature is obtained at the expense of the second and third layer temperatures which are slightly deteriorated (Fig. 2).

If $i = N$, the heat conduction between layer N and layer $N-1$ is assumed to be zero, i.e.,

$$F_{N-1} = 0 \tag{2.11a}$$

or

$$T_N = T_{N-1} \tag{2.11b}$$

Hence, the coefficients for the tridiagonal system of equations are given as follows

$$a_i = -1 \tag{2.12a}$$

$$b_i = 1 \tag{2.12b}$$

$$c_i = 0 \tag{2.12c}$$

$$r_i = 0 \tag{2.12d}$$

3. Cold Season/Region Processes

Snow and frozen soil are common land features in the cold seasons or regions. The previous force-restore framework in modeling the snowpack (Dickinson, 1988) is probably adequate to simulate the surface snow processes such as surface temperature and surface fluxes as used in the climate modeling studies (Yang et al., 1997). However, accurately simulating the timing of snowmelt and the profiles of soil temperatures may require an improved treatment of snowpack processes such as radiation attenuation and meltwater percolation within the snow layer, which may need multiple layers (Jin et al., 1999).

4. Future Development

The current treatment of soil moisture and temperature is probably one of the most comprehensive among the land-surface models for use in climate and weather studies. The current scheme needs extensive calibration and testing using the data from field sites and large river basin. Further development is still possible in model refinement, addition of extra processes, and improvement of numerical efficiency. Whether or not to include these refinements is largely dependent upon
the data availability and their impacts to climate/weather and biogeochemical processes to be modeled. Some possible areas of refinement are listed as follows.

(i) Water table and topography effects on soil moisture and runoff (Beven and Kirkby, 1979).
(ii) Soil water vapor transport and soil evaporation (Philip and de Vries, 1957).
(iii) Soil moisture movement in response to temperature gradients (Philip and de Vries, 1957; Milly and Eagleson, 1982; Bach, 1992).
(iv) Vertical soil heterogeneity (e.g., O, A, B, C horizons).
(v) Horizontal soil heterogeneity (e.g., different soil types in a grid box; Wetzel and Boone, 1995).
(vi) Soil water hysteresis (Lenhard et al., 1991).
(vii) Preferential flow (Germann, 1990).
(viii) Salute transport in the soil medium.
(ix) Efficiency, accuracy, and portability of the numerical schemes.

There is a rich literature in the soil science community which has developed methods and codes for the soil water transport (e.g., Simunek et al., 1994). Their work may serve as a basis for developing a soil model for the climate and weather related studies.

List of Figures

Fig. 1. Structure of the N-layer soil model (N=10). z is height above soil surface (treated as an interface), positive upward, whereas the layer index increases downward. All the moisture and heat fluxes are defined across the interfaces or boundaries, positive upward. The soil moisture, matric potential, temperature and heat capacity are the layer-averaged variables, defined at the center of the layer.

Fig. 2. Root-mean-square-error (RMSE) of temperatures between numerical (Section 2.2) and analytical (Appendix A) solutions for a 10-layer model, periodic forcing and 30-minutes time step. The results are from last day of a 20-day integration.

Appendix A: Analytical solutions for soil temperatures

For a homogeneous soil column, the heat diffusion equation is given by

$$c\frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial z^2} \tag{A.1}$$

with the boundary condition:

$$\left(-\lambda \frac{\partial T}{\partial z}\right)_{z=0} = F(0,t) \tag{A.2}$$

For a periodic forcing (e.g., diurnal forcing) at the surface,

$$F(0,t) = F_0 \cos(\omega t - \varepsilon) \tag{A.3}$$

one has the solution as follows,

$$T(z,t) = \overline{T} + \frac{F_0}{\sqrt{\omega c \lambda}} \exp(-kz)\cos(\omega t - \varepsilon - kz - \pi/4) \tag{A.4}$$

$$T(0,t) = \overline{T} + \frac{F_0}{\sqrt{\omega c \lambda}} \cos(\omega t - \varepsilon - \pi/4) \tag{A.5}$$

where $\overline{T}$ is the deep soil temperature when $z \to \infty$.

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