Physical Climatology Problem Set #2

(20 points) **1**. Hartmann Book, page 39 (Exercise # 2).

(20 points) **2**. Hartmann Book, page 39 (Exercise # 4).

(20 points) **3**. Hartmann Book, page 39 (Exercise # 5).

(20 points) **4**. Hartmann Book, page 39 (Exercise # 6).

(20 points) **5**. Astronomers sometimes determine the size of a star by a method that relies on the Stefan-Boltzmann Law. Determine the radius of the star Vapella from the following data: the flux of the starlight reaching the Earth is 1.2×10^{-8} Wm^{-2}, the distance of the star is 4.3×10^{17} m, and its surface temperature is 5200 K. Assume the star radiates like a blackbody.

Physical Climatology Homework #2
Answer Key

1. Page 39, #2

There are two methods of computing the solar "constant" for the young sun.

Method I:

$$S_{o\,young} = \frac{L_{o\,young}}{4\pi \bar{d}^2} = \frac{(1-0.3)\,L_o}{4\pi \bar{d}^2}$$

$$= \frac{(1-0.3) \times 3.9 \times 10^{26}}{4\pi [1.5 \times 10^{11}]^2}$$

$$\doteq 966 \ \mathrm{Wm^{-2}}$$

Method II:

$$S_{o\,young} = (1-0.3)\,S_o = (1-0.3) \times 1367$$

$$\doteq 957 \ \mathrm{Wm^{-2}}$$

The differences in both values may be due to rounding errors in Method I (in L_o and $\bar{d}$).

The emission temperature of Earth

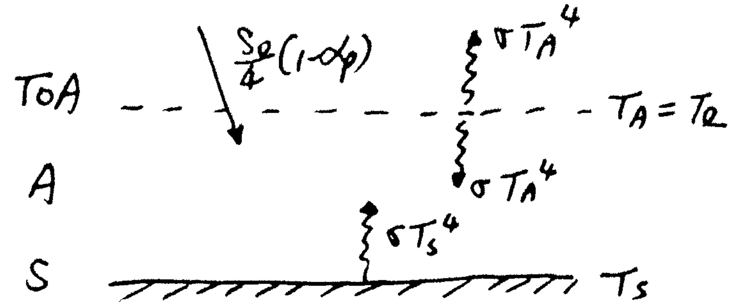
$$T_e = \left[\frac{\frac{S_{o\,young}}{4}(1-\alpha_p)}{\sigma} \right]^{\frac{1}{4}}$$

using $\alpha_p = 0.3$, $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-1}}$

$$T_e = \begin{cases} 233.67\ \mathrm{K} & \text{Method I} \\ 233.13\ \mathrm{K} & \text{Method II} \end{cases}$$

2. Page 39, #4

If the insolation is absorbed in the atmosphere, rather than at the surface, Fig 2.3 may be redrawn as below



The radiative energy balance for

ToA: $$\frac{S_0}{4}(1-\alpha_p) = \sigma T_A^4 \quad (1)$$

A: $$\frac{S_0}{4}(1-\alpha_p) + \sigma T_s^4 = 2\sigma T_A^4 \quad (2)$$

S: $$\sigma T_s^4 = \sigma T_A^4 \quad (3)$$

From (1): $T_A = \left[\frac{\frac{S_0}{4}(1-\alpha_p)}{\sigma}\right]^{\frac{1}{4}} = 255\ K$

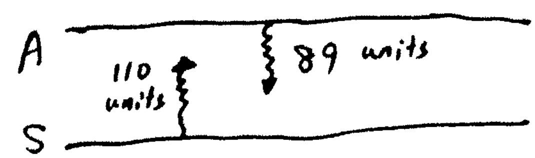
From (3): $T_s = T_A = 255\ K$

The model in Fig 2.3 gives $T_A = 255\ K$, $T_s = 303\ K$.

Conclusion: If the insolation is absorbed in the atmosphere, rather than at the surface, the surface temperature is MUCH LOWER. Actually, it is the same as the temperature at the top of the atmosphere, or as if there were NO atmosphere.

3. Page 39, #5

To estimate the blackbody temperature of the Earth's surface and the blackbody temperature of the atmosphere as viewed from the Earth's surface, we must know the upward emission of ~~Earth's surface~~ longwave radiation from the Earth's surface and the downward emission of longwave radiation from the atmosphere.



Longwave

(After Fig 2.4)

Fig 2.4 shows Earth emits 110 units longwave radiation and the atmosphere emits 89 units longwave radiation.

Assume Earth and the atmosphere are blackbody, we can use the ~~St~~ Stefan-Boltzmann Law. $E_{BB} = \sigma T^4$ $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

For Earth, $E_{BB} = 110$ units $= \frac{110}{100} \times 342\ Wm^{-2} = 376.2\ Wm^{-2}$

$$\therefore T_S = \left[\frac{E_{BB}}{\sigma}\right]^{1/4} = \left[\frac{376.2}{5.67 \times 10^{-8}}\right]^{\frac{1}{4}} = 285.4\,K \doteq 12.3\ ^\circ C$$

For the atmosphere. $E_{BB} = 89$ units $= \frac{89}{100} \times 342\ Wm^{-2} = 304.38\ Wm^{-2}$

$$\therefore T_A = \left[\frac{E_{BB}}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{304.38}{5.67 \times 10^{-8}}\right]^{\frac{1}{4}} = 270.7\,K \doteq -2.5^\circ C$$

Fig 2.3 shows that $T_S = 303\,K$, $T_A = 255^\circ K$

The lower T_S (by ~20 K) in Fig 2.4 is because

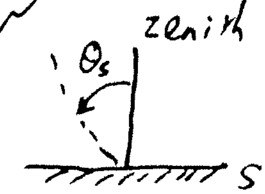
1. the atmosphere absorbs a portion of solar radiation,
2. turbulence & convection keep the surface cool.

The same reasons can also explain why T_A in Fig 2.4 is higher.

4. Page 39, #6.

The solar zenith angle is given by

$$\cos\theta_s = \sin\phi\sin\delta + \cos\phi\cos\delta\cosh \quad ①$$



where θ_s = the solar zenith angle; $0° \le \theta_s \le 90°$

(sunrise & sunset occurs when $\theta_s = 90°$
$\cos\theta_s < 0$ means the sun is below the horizon and the surface is in darkness)

ϕ = the latitude
- >0, the northern hemisphere
- $=0$ the equator
- <0 the southern hemisphere

δ = the declination angle, which can be computed from (A.6) in the Hartmann book, or from

$$\delta = 0.4093\sin\left[\frac{2\pi J}{365} - 1.405\right] \quad \text{in radians}$$

where J = the day of year, $= 172$ (June 21) summer solstice
$= 355$ (Dec 21) winter solstice

h = the hour angle relative to noon, which can be computed from

$$h = \frac{2\pi}{24}\times(t-12) \quad \text{in radians}$$

where t = hour of day, from 0 to 23 (or 1 to 24)

At Seattle, $\phi = 47°N = \frac{47}{180}\times\pi = 0.820304734$

at 9am, $h = \frac{2\pi}{24}\times(9-12) = -0.7853981 5$ (or 45°)

at Summer solstice, using the δ formula above or simply from the text, $\delta = 23.45° = \frac{23.45}{180}\times\pi = 0.409279702$

Substituting these values into ①, $\cos\theta_s = 0.747402$

or $\boxed{\theta_s = 42.82°}$

At winter solstice, $\delta = -23.45° = -0.409279702$,
$\cos\theta_s = 1.41884$, $\Rightarrow$ $\boxed{\theta_s = 81.29°}$

If $\phi = 32°N$ (Austin), then θ_s (summer solstice) $= 40.45°$, (winter solstice) $= 70.17°$

5. Assume r = the radius of Vapella

d = the Earth-Vapella distance

S_d = the flux density at d

E_{BB} = the flux density at r or Vapella's surface

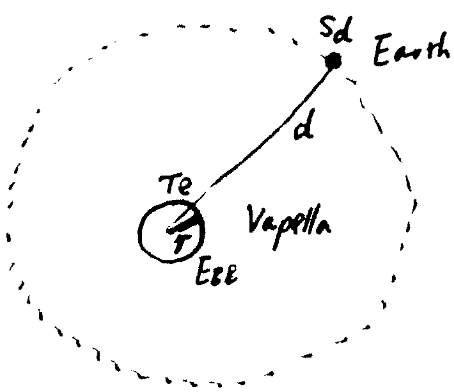
T_e = the emission temperature of Vapella

Conservation of energy requires

$$4\pi r^2 E_{BB} = 4\pi d^2 S_d \qquad (1)$$

(or Luminosity at r = Luminosity at d)

solving (1) for r gives



$$r = d\sqrt{\frac{S_d}{E_{BB}}} \qquad (2)$$

$$E_{BB} = \sigma T_e^4 \qquad (3)$$

substituting (3) into (2):

$$r = d\sqrt{\frac{S_d}{\sigma T_e^4}} = 4.3\times10^{17}\sqrt{\frac{1.2\times10^{-8}}{5.67\times10^{-8}\times5200^4}}$$

$$\doteq 7.3\times10^9\ m$$