HW #5 Physical Climatology

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1. From Table 5.1 (p.116),

water volume in groundwater & soil moisture $= 8.062 \times 10^6 \ km^3$
$= 8.062 \times 10^6 \times (1000 \ m)^3$
[volume]

From Figure 5.1 (p.116),

precipitation over land $= 75 \ cm / yr$, this is a depth unit per yr (per yr.). To do this, we need to convert it to the volume unit. we know land covers 30% of the Earth's surface.

Earth's radius $= 6.37 \times 10^6 \ m$ [see Appendix G p. 373]

Earth's surface area $= (6.37 \times 10^6 \ m)^2 \times \pi \times 4$

$\therefore$ The annual precip. over land in the volume unit $= 75 \times 10^{-2} \ m/yr \times 0.30 \times 4 \times 3.14 \times (6.37 \times 10^6 \ m)^2$

$$\text{The residence time} = \frac{\text{storage}}{\text{flux}}$$

$$= \frac{8.062 \times 10^6 \times (1000 \ m)^3}{75 \times 10^{-2} \ m/yr \times 0.30 \times 4 \times 3.14 \times (6.37 \times 10^6 \ m)^2}$$

$$= \frac{8.062 \times 10^{15}}{11467.03194 \times 10^{10}} \doteqdot \boxed{70 \ yr}$$

From Figure 5.1 (p.116), runoff from land $= 27 \ cm/yr$

Therefore, 10% of runoff $= 2.7 \ cm/yr$

It is equivalent to $\frac{2.7}{75} = 0.036$ of land precipitation.

$\therefore$ The residence time for 10% of runoff is

$$= \frac{70 \ yr}{0.036}$$

$$= \boxed{1944 \ yr}$$

2. From (4.32) latent heat flux is determined by

$$LE = \rho L\, C_{DE}\, U \left[q_s^*(1-RH) + RH \cdot Be^{-1} \frac{C_p}{L}(T_s - T_a) \right] \quad ①$$

The tle water vapor flux or evaporction can be obtained by multiplying L^{-1} on both sides, i.e.,

$$E = \rho\, C_{DE}\, U \left[q_s^*(1-RH) + RH\, Be^{-1} \frac{C_p}{L}(T_s - T_a) \right] \quad ②$$

The unit for LE is Wm^{-2}, for E $Kg\,m^{-2}\,s^{-1}$.

From (4.33), we have $Be^{-1} = \dfrac{L}{C_p} \dfrac{\partial q_s^*}{\partial T}$ ③

From (4.35), we have $\dfrac{\partial q_s^*}{\partial T} = q_s^* \dfrac{L}{R_v T^2}$ ④

Substituting ④ into ③: $Be^{-1} = \dfrac{L}{C_p} q_s^* \dfrac{L}{R_v T^2}$ ⑤

Substituting ⑤ into ②:

$$E = \rho\, C_{DE}\, U \left[q_s^*(1-RH) + RH \cdot q_s^* \frac{L}{R_v T_s^2}(T_s - T_a) \right] \quad ⑥$$

Given $C_{DE} = 10^{-3}$; $\rho = 1.2\ Kg\,m^{-3}$; $U = 5\ ms^{-1}$; $T_s - T_a = 2°C = 2K$

$L = 2.5 \times 10^6\ JKg^{-1}$; $R_v = 461\ JKg^{-1}K^{-1}$

(a) $T_s = 0°C = 273.15\ K$; $q_s^* = 3.75\ gKg^{-1} = 3.75 \times 10^{-3}\ Kg\,Kg^{-1}$; $RH = 50\% = 0.50$

Substituting these values into ⑥,

$$\therefore\ E = 1.2 \times 10^{-3} \times 5 \times \left[3.73 \times 10^{-3} \times (1-0.50) + 0.50 \times 3.75 \times 10^{-3} \times \frac{2.5 \times 10^6 \times 2}{461 \times 273.15^2} \right]$$

$= 1.28854 \times 10^{-5}\ Kg\,m^{-2}\,s^{-1}$ [water vapor flux]

$$= \frac{1.28854 \times 10^{-5}\ kg\,m^{-2}\,s^{-1}}{\underbrace{1000\ Kg\,m^{-3}}_{\text{density of water}}} \times \frac{100\ cm}{1\ m} \times \frac{86400\ s}{1\ day}$$

$= \boxed{0.111\ cm/day}$ [in depth of water per day]
easy to understand and use!

(b) $T_s = 0°C = 273.15\ K$; $q_s^* = 3.75\ g\,kg^{-1} = 3.75 \times 10^{-3}\ kg\,kg^{-1}$

$RH = 100\% = 1.00$

$$\therefore\ E = 1.2 \times 10^{-3} \times 5 \times \left[3.75 \times 10^{-3} \times (1-1.00) + 1.00 \times 3.75 \times 10^{-3} \times \frac{2.5 \times 10^6}{461 \times 273.15^2} \times 2\right]$$

$= 3.27076 \times 10^{-6}\ kg\,m^{-2}\,s^{-1}$

$= \boxed{0.028\ cm/day}$

(c) $T_s = 30°C = 303.15\ K$ $q_s^* = 27\ g\,kg^{-1} = 27 \times 10^{-3}\ kg\,kg^{-1}$

$RH = 50\% = 0.50$

$$\therefore\ E = 1.2 \times 10^{-3} \times 5 \times \left[27 \times 10^{-3} \times (1-0.50) + (0.50) \times 0\,27 \times 10^{-3} \times \frac{2.5 \times 10^6}{461 \times 303.15^2} \times 2\right]$$

$= 9.05596 \times 10^{-5}\ kg\,m^{-2}\,s^{-1}$

$= \boxed{0.78\ cm/day}$

(d) $T_s = 30°C = 303.15\ K$, $q_s^* = 27\ g\,kg^{-1} = 27 \times 10^{-3}\ kg\,kg^{-1}$

$RH = 100\% = 1.00$

$$\therefore\ E = 1.2 \times 10^{-3} \times 5 \times \left[27 \times 10^{-3} \times (1-1.00) + 1.00 \times 27 \times 10^{-3} \times \frac{2.5 \times 10^6}{461 \times 303.15^2} \times 2\right]$$

$= 1.91192 \times 10^{-5}\ kg\,m^{-2}\,s^{-1}$

$= \boxed{0.165\ cm/day}$

Conclusion: If the temperature is fixed, increasing RH from 50% to 100% results in a decreased evaporation rate by 75% (0°C) to 79% (30°C).

If RH is fixed, increasing temperature from 0°C to 30°C results in an increased evaporation rate by a factor of 5 (RH=100%) to 6 (RH=50%).

Relatively speaking, surface temperature is more important for determining E than RH.

3. Using the bulk aerodynamic formulas,

$$SH = C_p \rho C_{DH} U_r (T_s - T_a) \qquad (4.26)$$

$$LE = L \rho C_{DE} U_r (q_s - q_a) \qquad (4.27)$$

$$\doteq \rho L C_{DE} U_r \left[q_s^*(1-RH) + RH \cdot Be^{-1} \frac{C_p}{L} (T_s - T_a) \right] \qquad (4.32)$$

$$\therefore B_o \equiv \frac{SH}{LE} = \frac{C_p \rho C_{DH} U_r (T_s - T_a)}{\rho L C_{DE} U_r \left[q_s^*(1-RH) + RH \cdot Be^{-1} \cdot \frac{C_p}{L} (T_s - T_a) \right]}$$

If $C_{DH} = C_{DE}$

then

$$B_o = \frac{C_p}{L} \frac{T_s - T_a}{q_s^*(1-RH) + RH \cdot Be^{-1} \frac{C_p}{L} (T_s - T_a)} \qquad ①$$

Using

$$q_s^* = 0.622 \frac{e^*}{P}$$

$$= 0.622 \frac{6.11}{1000} \times \exp\left[\frac{L}{R_v} \left(\frac{1}{273} - \frac{1}{T_s} \right) \right] \qquad ②$$

$$Be^{-1} = \frac{L}{C_p} \frac{\partial q^*}{\partial T} = \frac{L}{C_p} \cdot q_s^* \cdot \frac{L}{R_v T_s^2} \qquad ③$$

where $C_p = 1004 \ J kg^{-1} K^{-1}$; $L = 2.5 \times 10^6 \ J kg^{-1}$

$R_v = 461 \ J kg^{-1} K^{-1}$

(a) $T_s = 0°C$, $RH = 70\% = 0.70$, $T_s - T_a = 2°C = 2K$
Then $q_s^* = 3.84 \times 10^{-3} \ Kg Kg^{-1}$, $Be^{-1} = 0.695363$, $\boxed{B_o = 0.52}$

(b) $T_s = 15°C$, $RH = 70\% = 0.70$, $T_s - T_a = 2°C = 2K$
then $q_s^* = 1.07997 \times 10^{-2} \ Kg Kg^{-1}$, $Be^{-1} = 1.75639$, $\boxed{B_o = 0.19}$

(c) $T_s = 30°C$, $RH = 70\% = 0.70$, $T_s - T_a = 2°C = 2K$
Then $q_s^* = 2.74 \times 10^{-2} \ Kg Kg^{-1}$, $Be^{-1} = 4.02687$, $\boxed{B_o = 0.077}$

4. ① High latitude land areas → low T_s, thus high B_o

② High B_o → low latent heat flux, or low evaporation.

③ Low E → high surface moisture content in the soil.

5. ① In the snow covered ground, the soil is generally wet.

② The ground can have a large capacity to hold snow, while the soil has a limited capacity to hold moisture.

③ In later spring, melt water from snow keeps the soil wet.

④ The snow melt consumes heat from the atmosphere.

⑤ the wet soil evaporates, which also cools the surface.

⑥ The summer soil moisture would be high and the temperature low.

However, if the winter & spring snowfall were replaced by rain showers, the excess precipitation (over evaporation) in winter would result in runoff. The soil would be drier. The August climate would be drier and hotter.

6. The bucket model of the land hydrology neglects

① the soil moisture transport within the soil layers,

② the vegetation control on Evapotranspiration,

③ the natural runoff production processes.

These are improved in the advanced land-surface models, which ① explicitly considers moisture movement within the soil layers

② represents the role of vegetation in regulating evapotranspiration, wind profiles, radiative balances,

③ accounts for various mechanisms of runoff production (e.g., infiltration excess, saturation excess), subgrid variability of precipitation and topography, etc.

7. (5.12) $E = \frac{1}{1+Be} E_{en} + \frac{Be}{1+Be} E_{air}$

where $E_{air} = \rho C_{DE} U (q_a^* - q_a)$

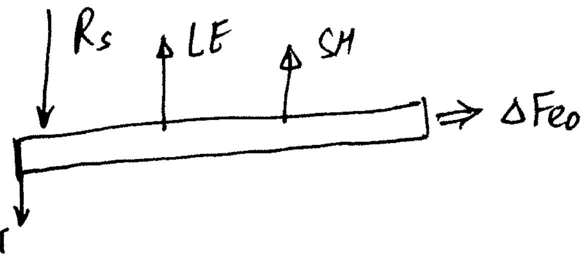
$= \rho C_{DE} U q_a^* (1 - RH)$

$E_{en} = \frac{1}{L}(R_s - \Delta F_{eo} - G)$

From (4.1), the surface energy balance is

$$G = R_s - LE - SH - \Delta F_{eo} \quad (1)$$

where



G: the storage in the surface layer

R_s: the net radiation

SH: the sensible heat flux

LE: the latent heat flux

ΔF_{eo}: the horizontal flux out of the surface layer

(2)

Let $B_o \equiv \frac{SH}{LE}$ (3)

Then $SH = B_o \cdot LE$

(3) into (1): $G = R_s - LE - B_o \cdot LE - \Delta F_{eo}$

$LE(1+B_o) = R_s - \Delta F_{eo} - G$ or $E(1+B_o) = \frac{1}{L}(R_s - \Delta F_{eo} - G)$

or

Let $E_{en} = \frac{1}{L}(R_s - \Delta F_{eo} - G)$ (4)

we have $E(1+B_o) = E_{en}$ (5)

Using the bulk aerodynamic formulas,

$$SH = C_p \rho C_{DH} U (T_s - T_a) \quad (6)$$

$$LE = L \rho C_{DE} U (q_s - q_a) \quad (7)$$

assuming $C_{DE} = C_{DH}$

Then $$B_o \equiv \frac{SH}{LE} = \frac{C_p}{L} \frac{T_s - T_a}{q_s - q_a} \quad (8)$$

If the surface is wet, $q_s = q^*(T_s) = q_s^*$

then $$B_o = \frac{C_p}{L} \frac{T_s - T_a}{q_s^* - q_a} = \frac{C_p}{L} \cdot \frac{T_s - T_a}{q_s^* - q_a^*} \cdot \frac{q_s^* - q_a^*}{q_s^* - q_a} \quad (9)$$

If the surface and the reference-level air temperatures are not too different, then $\dfrac{q_s^* - q_a^*}{T_s - T_a} \doteq \dfrac{\partial q^*}{\partial T}$

using $Be^{-1} \equiv \dfrac{L}{C_p} \dfrac{\partial q^*}{\partial T}$

(9) becomes $$B_o = Be \frac{q_s^* - q_a^*}{q_s^* - q_a} = Be \frac{(q_s^* - q_a) - (q_a^* - q_a)}{q_s^* - q_a}$$

$$= Be \left[1 - \frac{q_a^* - q_a}{q_s^* - q_a} \right] \quad (10)$$

(10) into (5): $$E(1 + B_o) = E_{en}$$

$$E + E \cdot Be = E_{en} + E\,Be \frac{q_a^* - q_a}{q_s^* - q_a} \quad (11)$$

From (7): $E = \rho C_{DE} U (q_s^* - q_a)$, $\quad q_s^* - q_a = \dfrac{E}{\rho C_{DE} U} \quad (12)$

(12) into (11): $$E + E\,Be = E_{en} + Be \frac{(q_a^* - q_a)}{\cancel{\rho C_{DE} U}} \cdot \rho C_{DE} \cdot U$$

$$\therefore \quad E = \frac{1}{1 + Be} E_{en} + \frac{Be}{1 + Be} E_{air}$$

where $E_{air} = \rho C_{DE} U (q_a^* - q_a) = \rho C_{DE} U q_a^* (1 - RH) \quad (13)$

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Physical Climatology
Homework 6 – Ann Thijs

1. The approximate volume of water retained in soil moisture and ground water is given in Table 5.1. Use the data in Figure 5.1. to calculate the time it would take for precipitation over land to deliver an amount of water equal to the soil water and groundwater. How long would it take to replace the groundwater and soil moisture if only 10% of the runoff could be redirected to replenishing the groundwater?

All groundwater and soil moisture add up to 8.062E06 km^3 of water (Table 5.1.). Divided over the total land surface of the Earth (148,300,000km^2, Wikipedia), this amounts to (8.062E06 km^3/148,300,000km^2=0.05436 km=) 5436 cm.
The precipitation over land is 75 cm/year (Figure 5.1), so it would take (5436 cm/75 cm.$year^{-1}$=) 72.48 years to deliver this amount of water. ✓
If only 10% of the runoff (total runoff = 27 cm/year; 10% =2.7 cm/year) would replenish soil moisture and groundwater, it would take (5436 cm/2.7 cm.$year^{-1}$=) 2013 years to replenish ✓ the soil moisture and ground water.

2. Use the bulk aerodynamic formula (4.32) to calculate the evaporation rate from the ocean, assuming that C_{DE} = 10E-3, U = 5m/s, and that the reference-level air temperature is always 2°C less than the sea surface temperature. Calculate the evaporation rate for (a-d). Assume a fixed density of 1.2kg/m^3. How would you evaluate the importance of relative humidity vs. the importance of surface temperature for determining the evaporation rate?

$E = \rho C_{DE} U(q_s^*(1-RH)+RH\, B_e^{-1} C_p(T_s-T_a)/L)$ **[units: kg/m^2.s = mm/s]**

$10^{-3} \Rightarrow$ 1.0e-3

ρ=1.2 kg/m^3 | U = 5m/s | T_s-T_a = 2 degrees C
C_{DE} = 10E-3 should be 1E-3 | RH = RH[%]/100 | C_p= 1004 J/K.kg
B_e^{-1}(0°C) = 1 | L = 2.5E6 J/kg
B_e^{-1}(30°C) = 1/0.2 =5 (Fig. 4.10)

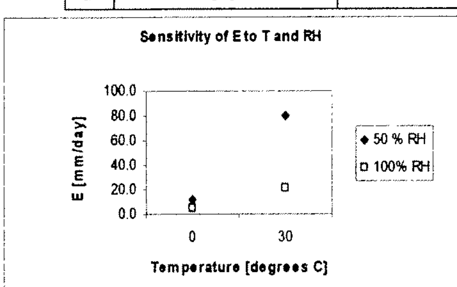
Since you are writing a code, why don't you calculate Be directly from

$$Be^{-1} = \frac{L}{C_p}\frac{\partial q^*}{\partial T} = \frac{L}{C_p}\cdot q^* \frac{L}{R_v}$$

(using eq 4.35)

	T_s [°C]	q_s* [kg/kg]	RH [%]	E [kg/m^2.s]	E [mm/day]
a	0	0.00375	50	0.0001366	11.8
b	0	0.00375	100	4.8192E-05	4.2
c	30	0.027	50	0.00093048	80.4
d	30	0.027	100	0.00024096	20.8

a factor 10 bigger



Both a decrease in relative humidity and an increase in temperature result in higher evaporation rates. At low temperatures, the evaporation rate is low – whatever the relative humidity is – due to a high equilibrium Bowen ratio (1 vs. 0.2). At high temperatures, the evaporation rate is high and strongly affected by the relative humidity. The relative humidity plays an important secondary role, because it provides the driving force for evaporation.

✓✗

3. Calculate the Bowen ratio using the bulk aerodynamic formulas for surface temperatures of 0, 15, and 30 degrees C, if the relative humidity of the air at the reference level is 70% and the air-sea temperature difference is 2 degrees C. Assume that the transfer coefficients for heat and moisture are equal.

$B_0 = SH/LE \approx C_p/L.((T_s-T_a(z_r))/(q_s-q_a(z_r)))$ (eq.1)

C_p= 1004 J/K.kg
~~L = 2.5E6 J/kg~~
Ts-Ta = 2°C

Assume $q_s \approx q_s^*$

$e^* = 6.11 \exp((L/R_v)((1/273)-(1/T[K])))$ (eq.2)
R_v = gas constant for water vapor = 461 J/K.kg
$q^* = 0.622\, e^*/p$ (eq.3)
p = 1000 mbar
$q_a = (RH/100)\, q_a^*$ (eq.4)

✓

$T_{surface}$	T_{air}	e_s^* (eq.2)	q_s^* (eq.3)	e_a^* (eq.2)	q_a^* (eq.3)	q_a (eq.4)	B_o (eq.1)	
0	−2	6.11	0.0038	7.06	0.0044	0.0031	**1.11**	0.52
15	~~17~~ 13	17.19	0.011	19.58	0.012	0.0085	**0.37**	0.19
30	~~32~~ 28	43.67	0.027	49.11	0.031	0.021	**0.14**	0.077

4. Use the results of problem 3 to explain why high latitude land areas often have high surface moisture content.

High latitude land areas correspond with low temperatures. In the table above, we can see that the Bowen ratio is larger at lower temperatures. A higher Bowen ratio (SH/LE) means more energy is lost from the surface by sensible heat than by latent heat. Or more simply said, at low temperatures, the surface does not lose as much energy by latent heat of vaporization, but more by sensible heat. As thus, the water in the soil will not evaporate as readily, which will lead to high surface moisture content.

5. Why is local winter and spring snow accumulation important for the summer soil moisture of midlatitude continental land areas? How do you think the August climate would change if the winter and spring snowfall were replaced by rainshowers?

The soil is assumed to have a fixed capacity to store moisture. If the sum of rainfall exceeds evaporation when the soil is saturated, runoff will occur at a rate to keep the soil saturated. When evaporation exceeds precipitation (and snowmelt), the amount of water in the soil drops.
There is also a maximum carrying capacity of the surface to hold ice and now, but this is very large. The snowcover lies on top of the soil and does not enter into the soil moisture balance unless it melts. The latent heat of melting must be supplied to the surface energy balance when melting occurs.

Both the large quantity of snow that can be stored on the surface, and the energy needed to melt the snow, result in higher soil moisture values later in the summer.

When snowfall would be replaced by rainshowers, the excess precipitation (over evaporation) in the wintertime would result in runoff. The soil would become drier from the moment that evaporation exceeds precipitation. This would give drier summer (August) soil conditions, resulting in lower evapotranspiration and overall a drier climate.

6. What are some shortcoming of the bucket model of land hydrology? How are these limitations addressed by more sophisticated models for land surface processes?

- The bucket model of land hydrology does not take the vegetative canopy, and the effect of the canopy on exchange processes into account. More sophisticated models incorporate the canopy and the following processes:
  - Coupling of momentum, heat and moisture budgets.
  - Rate of plant transpiration depends on PAR, temperature, relative humidity and availability of water.
  - Energy transfer through canopy to estimate leaf temperature.
  - VIS and NIR bands of solar radiation are treated differently.
  - Restricted airflow in canopy which impacts turbulent fluxes of momentum, heat and moisture.
  - Interception and evaporation from leaves.

- Number of soil layers
  - The number of soil layers can be increased in the bucket model of land hydrology and effects of low soil moisture on transpiration can be taken into account
  - In more sophisticated models, three soil layers are taken into account: (1) a thin surface layer from which evaporation occurs (2) a layer in which plant roots reside and from which water is drawn to provide for plant transpiration (3) a deeper layer, to which water is carried due to gravity and from which water can be drawn due to capillary action.

Ann Thijs Homework 6

question 7 : Derive 5.12 using the method outlined in the tex

1 Surface energy balance : ✓

$$G = Rs - LE - SH - \Delta Feo$$

G : energy storage
Rs : net radiative flux
LE : latent heat flux
SH : sensible heat flux
ΔFeo : horizontal flux out.

$$LE + SH = Rs - \Delta Feo - G$$

$$\downarrow \quad Bo = SH / LE$$

$$\underbrace{LE + Bo \cdot LE} = Rs - \Delta Feo - G$$

$$LE\,(1 + Bo) = Rs - \Delta Feo - G$$

$$E\,(1 + Bo) = 1/L\,(Rs - \Delta Feo - G) = Een$$

$$E\,(1 + Bo) = Een$$

2 bulk aerodynamic formulas: ✓

$$Bo = \frac{SH}{LE} = \frac{Cp \cdot \cancel{\rho} \cdot \cancel{CDH} \cdot \cancel{Vr} \cdot (Ts - Ta)}{L \cdot \cancel{\rho} \cdot \cancel{CDE} \cdot \cancel{Vr} \cdot (qs - qa)}$$

assume CDH = CDE

$$Bo = \frac{Cp}{L} \cdot \left(\frac{Ts - Ta}{qs - qa} \right)$$

$$dq^* / dT = (qs^* - qa^*) / (Ts - Ta)$$

$$Ts - Ta = (qs^* - qa^*) \cdot \left(\frac{dq^*}{dT} \right)^{-1}$$

$$Bo = \frac{Cp}{L} \cdot \left(\frac{qs^* - qa^*}{qs - qa} \right) \cdot \left(\frac{dq^*}{dT} \right)^{-1}$$

$$Be = \frac{Cp}{L} \left(\frac{\partial q^*}{\partial T} \right)$$

<eq. 4.33>

$$Bo = Be \left(\frac{qs^* - qa^*}{qs - qa} \right)$$

$$E(1+Be) = Een + E \cdot Be \cdot \left(\frac{qa^* - qa}{qs^* - qa} \right)$$

$$E(1+Be) = Een + Be \cdot (qa^* - qa) \cdot \rho \cdot C_{DE} \cdot U$$

$$E = \frac{1}{(1+Be)} Een + \frac{Be}{(1+Be)} \cdot \underbrace{\rho \cdot C_{DE} \cdot U \cdot (qa^* - qa)}_{\text{define as } Eair}$$

$$\boxed{E = \frac{1}{(1+Be)} \cdot Een + \frac{Be}{(1+Be)} \cdot Eair}$$

Excellent!