Reynolds Convention: Any process $S$ can be written as

$$S = \bar{S} + S'$$

mean fluctuation

with the following properties

$$\bar{S'} = 0$$

$$\bar{S W} = \bar{S} \bar{W} + \bar{S W}'$$

$$u + v = u' + v'$$

$$\alpha S = \alpha \bar{S}$$ (a = constant)

$\bar{S W}'$ = covariance

$\bar{S}$ = the 1st order moment of $S$

$\bar{S'^2}$ = the 2nd order moment of the joint process $S$ and $W$

Governing Equations for Atmospheric Flow & Garrett, p.20-26

The continuity equation: conservation of mass

$$\frac{1}{\rho} \frac{d \rho}{dt} + \frac{\partial u_i}{\partial x_j} = 0$$

The equation of state: the gas law

$$p = \rho R_W T$$ or $$p R d T$$

The thermodynamic equation: Conservation of enthalpy (or sensible heat) per unit mass $C_p\theta$

$$\frac{d (C_p\theta)}{dt} = \kappa \frac{\partial^2 (C_p\theta)}{\partial x_i^2} + \rho \frac{\partial F_j}{\partial x_j}$$

$F_j$ = radiative heat flux

The Navier-Stokes equations: conservation of momentum

$$\frac{du_i}{dt} = \nu \nabla^2 u_i - \frac{\partial p}{\partial x_i} - g_i e_{ijk} \frac{\partial \theta}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

The humidity equation: conservation of water vapour

$$\frac{d \theta}{dt} = k_j \frac{\partial \theta}{\partial x_j}$$
Turbulent Fluxes:

Comparing with \[ \frac{\partial \bar{q}}{\partial t} + u_j \frac{\partial \bar{q}}{\partial x_j} = K_\rho \frac{\partial^2 \bar{q}}{\partial x_j^2} \]

the mean humidity equation \[ \frac{\partial \bar{q}}{\partial t} + u_j \frac{\partial \bar{q}}{\partial x_j} = K_\rho \frac{\partial^2 \bar{q}}{\partial x_j^2} - \frac{\partial \bar{u} \bar{q}'}{\partial x_j} \]

contains a new term \( \frac{\partial \bar{u} \bar{q}'}{\partial x_j} \rightarrow \) flux divergence, which act as source terms to change the mean concentration, and which are direct result of the non-linear terms \( u_j \frac{\partial \bar{q}}{\partial x_j} \) in the instantaneous equations.

\[ \begin{align*}
\underline{u_j \bar{q}'} & \quad \text{turbulent moisture flux} \\
\underline{u_j \bar{q}'} & \quad \text{turbulent heat flux} \\
\underline{u_j \bar{u_j}} & \quad \text{turbulent momentum} \rightarrow [\text{Reynolds stress}] 
\end{align*} \]

In ABL, we are only concerned with \( p\bar{u} \bar{q}', p\bar{w} \bar{q}', p \bar{u} \bar{w} \) [vertical fluxes].

Closure Problem: averaging the equations leads to a situation where there are more unknowns than equations.

Parameterization: representation of the unknown fluxes using resolvable mean variables.

K-closure (1st-order) closure

\[
\begin{align*}
\tau_x &= -p \bar{u} \bar{w}' = \rho K_m \frac{\partial \bar{u}}{\partial y} \\
\tau_y &= -p \bar{v} \bar{w}' = \rho K_m \frac{\partial \bar{v}}{\partial y} \\
\tau_z &= -p \bar{w} \bar{w}' = \rho K_m \frac{\partial \bar{w}}{\partial z} \\
E &= p \bar{u} \bar{q}' = -p K_m \frac{\partial \bar{q}}{\partial y}
\end{align*}
\]
EXERCISE IV

Deriving governing equations for mean quantity (conservation of water vapor)

For the instantaneous specific humidity \( q \), in the absence of phase changes,

\[
\frac{d\bar{q}}{dt} = kv \frac{\partial \bar{q}}{\partial x_j}
\]

or

\[
\frac{\partial \bar{q}}{\partial t} + u_j \frac{\partial \bar{q}}{\partial x_j} = kv \frac{\partial \bar{q}}{\partial x_j}
\]

Using the Reynolds decomposition, \( q = \bar{q} + q' \)

\[ u_j = \bar{u}_j + u'_j \]

\[
\frac{\partial (\bar{q} + q')}{\partial t} + (\bar{u}_j + u'_j) \frac{\partial (\bar{q} + q')}{\partial x_j} = kv \frac{\partial^2 (\bar{q} + q')}{\partial x_j^2}
\]

\[
\frac{\partial \bar{q}}{\partial t} + \bar{u}_j \frac{\partial \bar{q}}{\partial x_j} + \bar{u}'_j \frac{\partial \bar{q}}{\partial x_j} + u_j \frac{\partial q'}{\partial x_j} + u'_j \frac{\partial q'}{\partial x_j} = kv \frac{\partial^2 \bar{q}}{\partial x_j^2} + kv \frac{\partial^2 q'}{\partial x_j^2}
\]

Applying \(-\) on both sides, note

\[
\bar{q} = \bar{q}, \quad \bar{u}_j = 0, \quad q' = 0, \quad \frac{\partial \bar{q}}{\partial t} = 0, \quad \frac{\partial^2 \bar{q}}{\partial x_j^2} = 0
\]

\[
\bar{u}_j = \bar{u}_j, \quad \frac{\partial q'}{\partial x_j} = 0
\]

The ABL flow is assumed incompressible,

\[
\frac{\partial \bar{u}_j}{\partial x_j} = \frac{1}{\rho} \frac{\partial p}{\partial x} = 0
\]

\[
\frac{\partial \bar{u}_j}{\partial x_j} = 0, \quad \bar{u}_j = 0 \quad \frac{\partial q'}{\partial x_j} = 0 \quad \bar{u}_j \frac{\partial q}{\partial x_j} = \frac{\partial (\bar{u}_j \bar{q})}{\partial x_j} + \frac{\partial q}{\partial x_j}
\]

\[
\frac{\partial \bar{q}}{\partial x_j} + \bar{u}_j \frac{\partial q}{\partial x_j} + \frac{\partial q}{\partial x_j} = kv \frac{\partial \bar{q}}{\partial x_j}
\]
For the horizontally homogeneous ABL, the mean equations

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} - \frac{\partial \bar{w}}{\partial z} \\
\frac{\partial \bar{v}}{\partial t} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} - \frac{\partial \bar{v}}{\partial z} \\
\frac{\partial \bar{w}}{\partial t} &= \frac{1}{\rho \gamma} \frac{\partial \bar{p}}{\partial z} - \frac{\partial \bar{w}}{\partial z} \\
\frac{\partial \bar{\theta}}{\partial t} &= -\frac{\partial \bar{w}}{\partial z}
\end{align*}
\]

\[
f = 2 \pi \sin \phi \quad \text{Coriolis parameter}
\]

\[
\Omega = 7.29 \times 10^{-5} \, \text{rad s}^{-1} \quad \text{the angular velocity}
\]

\[
\phi = \text{latitude}
\]

\[
f > 0 \quad \text{N.H.}
\]

\[
f < 0 \quad \text{S.H.}
\]

For steady-state conditions, \( \frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{v}}{\partial t} = 0 \)

the momentum equations

\[
\begin{align*}
0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} - \frac{\partial \bar{w}}{\partial z} \\
0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} - \frac{\partial \bar{v}}{\partial z}
\end{align*}
\]

Above the ABL, the turbulent terms are zero

\[
\begin{align*}
\bar{u} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} \quad &\Rightarrow \text{Geostrophic Balance} \\
0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u}
\end{align*}
\]

pressure gradient terms Coriolis terms
The Geostrophic Wind

\[ U_g = -\frac{f}{\frac{\partial p}{\partial y}} \]
\[ V_g = \frac{1}{f} \frac{\partial p}{\partial x} \]

If \( f = 10^{-4} \text{ s}^{-1} \)

\[ \rho = 1 \text{ Kg m}^{-3} \]

\[ \frac{\partial p}{\partial y} = 1 \text{ mb/(100 Km)} \]

\[ U_g \approx \frac{1}{10^{-4} \times 1} \times \frac{1 \times 10^2 \text{ Pa}}{100 \times 1000 \text{ m}} \]

\[ = 10 \text{ m s}^{-1} \]
Ekman Spiral: A useful 1st approximation to the neutral wind profile above the surface layer.

Consider the momentum equations in the steady-state case:

\[ \begin{align*}
\frac{\partial}{\partial z} \left( \begin{array}{c}
u \nu' \omega' \\
\omega \end{array} \right) & = -f(\nu - V_g) \\
\frac{\partial}{\partial z} \left( \begin{array}{c}
u \nu' \omega' \\
\omega \end{array} \right) & = f(U - U_g)
\end{align*} \]

\[-U' \omega' = k_m \frac{\partial \nu}{\partial z} \]

\[-\nu' \omega' = k_m \frac{\partial \omega}{\partial z} \]

\[\int k_m \frac{\partial \nu}{\partial z} = -f(\nu - V_g) \]

\[\int k_m \frac{\partial \omega}{\partial z} = f(U - U_g) \]

Let
\[\frac{\nu}{V_g} = U + i \nu \]
\[\nu = \begin{cases} U_g & z = 0 \\ V_g & z = \infty \end{cases} \]

\[ \begin{align*}
\frac{\partial^2}{\partial z^2} (U + i \nu) & = \frac{i f}{k_m} \left[ (U - U_g) + (\nu - V_g) i \right] \\
\frac{\partial^2}{\partial z^2} \nu & = \frac{i f}{k_m} \left( U - V_g \right) \\
\nu & = V_g \exp(-a_0 \nu)
\end{align*} \]

\[ a_0 = \sqrt{\frac{f}{k_m}} \]

\[ \frac{\nu}{V_g} = \begin{cases} U_g & z = \frac{\pi}{2} \\ V_g & z = \pi \end{cases} \]

\[ \begin{align*}
\theta_0 & = 0, \quad U = V = 0 \\
\theta & = \frac{\pi}{2}, \quad U = V_g, \quad V = V_g \exp\left(-\frac{\pi}{2}\right) \\
\theta & = \pi, \quad U = V_g \exp\left(-\frac{\pi}{2}\right), \quad V = 0
\end{align*} \]
Roughness length:  

\[ L_r = -\frac{\tau}{\rho u_*} \]

\[ = \frac{\tau}{\rho u_*^2} \]

\[ = \frac{\tau}{\rho K M \frac{\partial u}{\partial y}} \]

If  \( K M = (x^2) \frac{\partial u}{\partial y} \) then  \( \frac{\partial u}{\partial y} = \frac{u_*}{k^2} \)

\[ \int_0^u \frac{du}{d\bar{u}} = \int_0^z \frac{u_*}{k^2} d\bar{z} \]

\[ \bar{u} = \frac{u_*}{K} \ln \frac{z}{z_0} \]

The logarithmic wind profile is generally true for ABL under neutral conditions.

\( z_0 = \) the roughness length, a constant of integration close to so that \( \bar{u} = 0 \) at \( z = z_0 \)

\( K = \) a \( \approx 0.4 \) universal constant, \( \approx 0.4 \)

\[ z_0 = \begin{cases} 1 \text{ mm} & \text{for average seas to more than 1 m} \\ 1-4 \text{ cm} & \text{for grassy fields} \\ 0.1h_c & \text{[between 0.07 and 0.14 h_c]} \end{cases} \]

\( h_c = \) height of Canopy

Zero-plane displacement \( d \) introduced to retain the logarithmic form of the profile in neutral conditions for measurement made above tall vegetation.

\[ K \frac{\partial u}{\partial y} = \ln \frac{z-d}{z_0} \]
Various forms of latent heat flux:

\[ LE = LP \ \frac{w'\theta'}{\theta} \] (Covariance)

\[ = LP CE U (\theta_s - \theta) \] (Bulk transfer relations)

\[ = LP \ \frac{\theta_s - \theta}{\gamma_e} \] (Aerodynamic resistance relations)

\[ = LP \ \frac{\theta_s (T_f) - \theta}{\gamma_s} \] (Surface resistance)

\[ = \beta E \ E_p \] (Potential evaporation, \( E_p = \frac{\theta_s (T_f) - \theta}{\gamma_e} \))

\[ = \Gamma^* (\gamma_e - \theta) + (1 - \Gamma^*) \frac{\theta}{\gamma_e} \frac{PL \delta \theta}{\gamma_e} \] (Pease Monteith Equation)

\[ \text{Where:} \]

\[ \Gamma^* = \frac{S}{S + \gamma_s} \]

\[ 1 - \Gamma^* = \frac{\gamma_e}{S + \gamma_s} \]

\[ \gamma^* = \gamma \frac{\gamma_e + \gamma_s}{\gamma_e} = \gamma \left(1 + \frac{\gamma_s}{\gamma_e}\right) \]

\[ S = \frac{\gamma^*}{\gamma} \quad \gamma = \frac{C_p}{L} \]

\[ q^* = 0.622 \frac{q^*}{\rho} \]

\[ \frac{\partial q^*}{\partial T} = 0.622 \frac{\partial q^*}{\partial T} = q^* \frac{L}{K \sqrt{T}^2} \] (See Eqn 4.35)

\[ \gamma_s = \frac{7.04 c}{\text{LAI}} \quad \text{LAI} = \frac{\Sigma L A_i C_i}{A} \]
Surface energy balance \( R_n - G_o = H + LE \)
\[ R_n = \text{SW}_i - \text{SW}_o + \text{LW}_i - \text{LW}_o \]
\[ = \text{SW}_i (1 - \alpha) + \varepsilon (\text{LW}_i - \sigma T_e^4) \]

Soil temperature and heat flux
\[ \frac{\partial (\rho_s C_s T_s \Delta z)}{\partial t} = -\frac{\partial G}{\partial z} \]
\[ \Delta z \to 0, \quad \rho_s C_s \Delta z \frac{\partial T_s}{\partial t} = -\frac{\partial G}{\partial z} \]
\[ G = -K_s \frac{\partial T_s}{\partial z} \]
\[ K_s = \rho_s C_s \frac{\partial T_s}{\partial z} \]

\[ \therefore \frac{\partial T_s}{\partial t} = K_s \frac{\partial^2 T_s}{\partial z^2} \]

Consider diurnal forcing,
\[ T_s|z=0 = T_0 = \bar{T} + A_0 \sin(\omega t) \]
\[ T_s|z=\infty = \bar{T} \]

Solution:
\[ T_s = \bar{T} + A_0 \exp\left(-\frac{z}{D}\right) \sin(\omega t - \frac{z}{D}) \]
\[ G_o = \rho_s C_s (K_s \Delta z) \frac{1}{2} A_0 \sin(\omega t + \frac{\pi}{2}) \]

So the soil heat flux is \( \frac{\pi}{2} \) out of phase with the temperature wave. Maximum \( G_o \) occurs 3 hours before the maximum surface temperature.
A key variable linking the surface energy balance (Ch. 4) and the water balance (Ch. 5) is evaporation.

In Ch. 4, the latent heat flux \( LE \), \( \text{Wm}^{-2} \)

In Ch. 5, evaporation \( E \), \( \text{Kg m}^{-2}\text{s}^{-1} \) (water vapor flux)

Both \( LE \) and \( E \) are linked by the latent heat of vaporization, \( L \).

\[
L = 2.5 \times 10^6 \text{ J kg}^{-1} \text{ at } 0^\circ \text{C} \\
= 2.25 \times 10^6 \text{ J kg}^{-1} \text{ at } 100^\circ \text{C}
\]

\[
\therefore \quad LE = L \times E
\]

SI unit for \( E \) is \( \text{Kg m}^{-2}\text{s}^{-1} \), but a commonly used unit in hydro-climatology for water fluxes is \( \text{mm/day} \).

Below is an example for this conversion:

If \( E = 1.28854 \times 10^{-5} \text{ Kgm}^{-2}\text{s}^{-1} \) (water vapor flux)

\[
= 1.28854 \times 10^{-5} \text{ Kgm}^{-2}\text{s}^{-1} \times \frac{1000 \text{ mm}}{1 \text{ m}} \times \frac{86400 \text{ s}}{1 \text{ day}}
\]

\[
= 1.11 \text{ mm/day} \quad \text{[in depth of water per day]}
\]

This \( \text{mm/day} \) is also used to describe precipitation, runoff.
\[
SH = C_p^p C_d U (T_s - T_a) \quad \frac{dSH}{dT_s} = C_p^p C_d U
\]

\[
B_0 = \frac{SH}{LE} \quad \begin{cases} 
\text{If the surface is wet, and} \\
\text{the air is saturated, and assuming} \\
C_d H = C_d E, \text{ we have} \\
B_0 = \frac{SH}{LE} = \frac{C_p^p (T_s - T_a)}{L \left( \frac{g_s}{g_d} - \frac{g_e}{g_d} \right)} \\
= Be \end{cases}
\]

\[
\text{Combination equation (or Perman equation) for } E
\]

- Wet surface → surface air is saturated, \(B_0 = Be \left[ 1 - \frac{g_s}{g_s} \right] \)
- Bulk aerodynamic formula \( E = \rho C_d U (g_s - g_a) \)

\[
E = \rho C_d U (g_s - g_a) = \rho C_d U \left( \frac{g_s}{g_s} - g_a \right)
\]

\[
\text{Surface energy balance} \quad R_s - LE - SH - G = 0
\]

\[
E_{en} = \frac{1}{L} (R_s - G) = E (1 + B_0)
\]

\[
E(1 + Be) = E_{en} + E Be \frac{g_s - g_a}{g_s - g_a}
\]

\[
\begin{array}{c}
\text{Using the bulk formula for } E \text{ to eliminate } E \text{ on the right hand side,} \\
E = \frac{1}{1 + Be} E_{en} + \frac{Be}{1 + Be} E_{air}
\end{array}
\]

\[
\begin{array}{c}
\text{Available energy for turbulent exchange} \\
= f(R_s, g)
\end{array}
\]

\[
\begin{array}{c}
\text{Atmospheric demand} \\
f(RH, U, T)
\end{array}
\]

\[
\text{Atmospheric demand at one level in the lower atmosphere}
\]
Linearization in the surface energy balance

The blackbody emission from the surface can be linearized about air temperature at the reference level,

\[ \sigma T_s^4 = \sigma T_a^4 + 4 \sigma T_a^3 (T_s - T_a) + \ldots \]

\[ \frac{\partial (\sigma T_s^4)}{\partial T_s} = 4 \sigma T_a^3 \]

Over water or wet land surfaces, the specific humidity at the surface is equal to the saturation specific humidity, \( q^* \), at the temperature of the surface

\[ q_s = q^*(T_s) = q^*_s \]

The specific humidity of saturated air at the reference height can be approximated with a first-order Taylor series

\[ q_a^* = q^*(T_a) = q^*_s + \frac{\partial q^*_s}{\partial T} |_{T_s} (T_a - T_s) + \ldots \]

Using

\[ \frac{q_a}{q_a^*} = \frac{q_a}{q^*_s} = RH \]

\[ q_a = RH \cdot q_a^* = RH \left[ q^*_s (T_s) + \frac{\partial q^*_s}{\partial T} |_{T_s} (T_a - T_s) \right] \]

\[ q_s - q_a = q_s - RH \cdot q_a = q_s - q_s (1 - RH) - RH \frac{\partial q^*_s}{\partial T} |_{T_s} (T_a - T_s) \]

\[ \text{LE} = \int \rho c_p E \left[ q_s - q_a \right] = \int \rho c_p E \left[ q^*_s (1 - RH) - RH \frac{\partial q^*_s}{\partial T} |_{T_s} (T_a - T_s) \right] \]

\[ \text{LE} = \rho c_p E \left[ q^*_s (1 - RH) + RH \frac{\partial q^*_s}{\partial T} |_{T_s} (T_a - T_s) \right] \]

where

\[ \text{Be}^{-1} = \frac{L}{c_p} \left. \frac{\partial q_s}{\partial T} \right|_{T_s} = \frac{L}{c_p} q^*_s \frac{L}{R_v T_s^2} \]
When the surface is not wet, need to consider the surface resistance.

Peotman-Monteith equation.

These equations are useful for measurements or for calculations when data input is limited.

More elaborate methods as used in LSMs to account for more physical processes, thereby requiring more data input. See my review of LSMs (a powerpoint presentation) and Dr. Fei Chen's overview. (and a review article)
Analytical solution for $LCL$

$$RH = \frac{e}{e^*}$$

$$\ln RH = \ln e - \ln e^*$$

$$d \ln RH = d \ln e - d \ln e^* \quad (1)$$

$$\therefore \quad e = \frac{(\gamma - 1)R}{0.622 + 0.378 \, f}$$

$$\therefore \quad \ln e = \ln \left( \frac{\gamma}{0.622 + 0.378 \, f} \right) + \ln P$$

For an adiabatic process, $\Theta = \text{Const}$, $\gamma = \text{Const}$.

$$d \ln e = d \ln P$$

$$\therefore \quad C_p \, d \ln \Theta = C_p \, d \ln T - R_d \, d \ln P = 0$$

$$\therefore \quad \frac{C_p}{R_d} \, d \ln T = d \ln P = d \ln e \quad (2)$$
\[ e^* = 6.11 \exp \left\{ \frac{L}{R_v} \left( \frac{1}{273.15} - \frac{1}{T} \right) \right\} \]

\[ \ln e^* = \ln 6.11 + \frac{L}{R_v} \left( \frac{1}{273.15} - \frac{1}{T} \right) \]

\[ d \ln e^* = + \frac{L}{R_v T^2} \, dT \]

\[ = \frac{L}{R_v T} \, d \ln T \]

Substituting (3) and (2) into (1):

\[ \int_{\ln RH}^{T} \, d \ln RH = \frac{C_p}{R_d} \ln \frac{T}{T} - \frac{L}{R_v} \ln \frac{T}{T} \]

\[ = \int_{T}^{T} \left( \frac{C_p}{R_d} - \frac{L}{R_v} \right) d \ln T \]

\[ - \ln RH = \frac{C_p}{R_d} \ln \frac{T}{T} + \frac{L}{R_v} \left( \frac{1}{T} - \frac{1}{T} \right) \]

\[ \text{approx:} \quad T_* = \frac{1}{\frac{1}{T-55} - \frac{\ln RH}{2840}} + 55 \]

\[ T_*, T \text{ in K} \]
\[ \ln \frac{T^*}{T} = \frac{Rd}{C_p} \ln \frac{P^*}{P} \]

\[ P^* = P\left(\frac{T^*}{T}\right)^{\frac{C_p}{Rd}} \]

\[ T, \quad e^*, \]
Rainfall, averaged over the earth's surface, is about 1 m y\(^{-1}\), or 0.275 cm day\(^{-1}\). On the basis of this information, calculate the following:

(a) The average upward flux of water vapor at the Earth's surface.

\[
E = \frac{P}{S} \quad \text{(i.e. evaporation = precipitation)}
\]

= 0.275 cm day\(^{-1}\)

= 0.275 \times 10 \text{ mm} \times \left( \frac{86400 \text{ s}}{\text{day}} \right)

= 3.18 \times 10^{-5} \text{ mm s}^{-1}

= 3.18 \times 10^{-5} \text{ Kg m}^{-2} \text{ s}^{-1}

Note that for water vapor flux, in water equivalent:

\[
1 \text{ mm} = 1 \text{ Kg m}^{-2}
\]

Thus, to say, 1 Kg of water, if spread out over 1 m\(^2\) of area, will have 1 mm thickness. Because 1 kg/(1000 kg m\(^{-3}\) m\(^3\)) = \(10^{-3}\) m = 1 mm

(b) The resulting energy flux.

\[
\text{latent heat of vaporization} \ L = 2.5 \times 10^6 \text{ Jkg}^{-1}
\]

\[
LE = 2.5 \times 10^6 \text{ Jkg}^{-1} \times 3.18 \times 10^{-5} \text{ Kg m}^{-2} \text{ s}^{-1}
\]

= 79.5 J m\(^{-2}\) s\(^{-1}\)

= 79.5 \text{ Wm}^{-2}

(c) How does this flux compare with the globally averaged incoming solar radiation?

The globally averaged incoming solar radiation is \(\frac{S_0}{4} = \frac{1367}{4} = 341.8 \text{ Wm}^{-2}\).

\[
\frac{LE}{S_0/4} = \frac{79.5}{341.8} \approx 23\%
\]

The answer is 23% of it.

(d) If this heat were uniformly distributed within the layer from 900 to 300 mb, what would be the resulting heating rate?

\[
\frac{dT}{dt} = \frac{LE}{C_a} \quad \text{where} \quad \bar{C}_a = \frac{\Delta P}{g} \cdot C_P = \frac{(900-300) \times 100 \text{ mb}}{g} \times 0.026 \times 1004 J \text{K}^{-1}
\]

\[
\bar{C}_a = 6146838.8 \text{ Jm}^{-2} \text{K}^{-1}
\]

\[
\frac{dT}{dt} = \frac{LE}{\bar{C}_a} = \frac{79.5}{6146838.8} \text{ Jm}^{-2} \text{K}^{-1} = 1.293326673 \text{ Ks}^{-1} \approx 1.12 \text{ K day}^{-1}
\]

= 1.12 \text{ °C day}^{-1}
Quiz 4

(a) Globally and annually, rainfall should be balanced by the water supply to the atmosphere. This is because the atmospheric water storage is so small, with a residence time of only 10 days. Therefore, we can say

\[ E = \rho = 1 \text{ m/year} = 1 \text{ m} \times \frac{1000 \text{ kg}}{1 \text{ m}^3} \times \frac{1}{1 \text{ year}} \]

Proof: Eq. 5.1:

\[ P = 107 \times 0.7 + 75 \times 0.3 \quad \text{[cm]} \]
\[ E = 188 \times 0.7 + 48 \times 0.3 \quad \text{[cm]} \]
\[ = 10^3 \text{ Kg m}^{-2} \text{ year}^{-1} \]

(b) Latent heat flux

\[ = E \times \text{latent heat of vaporization} \]
\[ = E \times L \]
\[ = 10^3 \text{ Kg m}^{-2} \text{ year}^{-1} \times 2.5 \times 10^6 \text{ J Kg}^{-1} \]
\[ = 2.5 \times 10^9 \text{ J m}^{-2} \text{ year}^{-1} \]
\[ = 2.5 \times 10^9 \text{ J m}^{-2} \text{ year}^{-1} \times \frac{1}{365 \times 24 \times 60 \times 60} \text{ s}^{-1} \]
\[ = 2.5 \times 10^4 \text{ W m}^{-2} \]

(c) \[ \frac{\text{LE}}{50/4} = \frac{2.5 \times 10^9 / (365 \times 24 \times 60 \times 60)}{1367/4} \text{ W m}^{-2} \]

(d) \[ \Delta m = \frac{\Delta p}{9} = \frac{(900-300) \text{ mb} \times 100 \text{ Pa}}{1 \text{ mb}} \frac{1}{9.8 \text{ m s}^{-2}} \]
\[ \overline{C_a} = \Delta m \cdot C_{pa} = \frac{\Delta p}{9} \cdot C_{pa} \]
where \( C_{pa} = 1004 \text{ J kg}^{-1} \text{ K}^{-1} \)

Energy input = LE, this will cause

heat storage change \( \overline{C_a} \frac{\Delta T}{\Delta t} \)

2nd Law of Thermodynamics,

\[ \frac{\Delta T}{\Delta t} = \frac{\text{LE}}{\overline{C_a}} = \frac{2.5 \times 10^9}{365 \times 24 \times 60 \times 60} \times \frac{1}{300 \text{ mb} \times (900-300) / 100 \times 1004} \]