

Reynolds Convention: Any process S can be written as

$$S = \bar{S} + S'$$

mean fluctuation

with the following properties

$$\left\{ \begin{array}{l} \overline{S'} = 0 \\ \overline{S'W'} = \bar{S} \bar{W} + \overline{S'W'} \\ \overline{u+v} = \bar{u} + \bar{v} \\ \overline{aS} = a \bar{S} \quad (a = \text{constant}) \end{array} \right.$$

$\overline{S'W'}$ = covariance

\bar{S} = the 1st-order moment of S

$\overline{S'^2}$ = the 2nd- " " " "

$\overline{S'W'}$ = the 2nd- " " of the joint process S and W

Governing Equations for Atmospheric Flow [Garratt, p.20-26] $\frac{d}{dt} = \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$

The continuity equation: conservation of mass

$$\frac{1}{\rho} \frac{d\rho}{dt} + \frac{\partial u_j}{\partial x_j} = 0$$

The equation of state: the gas law

$$p = \rho R_w T \quad \text{or} \quad p R_d T_v$$

The thermodynamic equation: Conservation of enthalpy (or sensible heat) per unit mass, $C_p \theta$.

$$\frac{d(C_p \theta)}{dt} = K_T \frac{\partial^2 (C_p \theta)}{\partial x_j^2} + \frac{1}{\rho} \frac{\partial R_j}{\partial x_j} \quad R_j = \text{Radiative heat flux}$$

The Navier-Stokes equations: conservation of momentum

$$\frac{du_j}{dt} = \rho^{-1} \frac{\partial p}{\partial x_j} - g \delta_{j3} - 2 \rho^{-1} \epsilon_{ijk} \eta_j u_k + \nu \frac{\partial^2 u_j}{\partial x_j^2}$$

The humidity equation: Conservation of water vapour

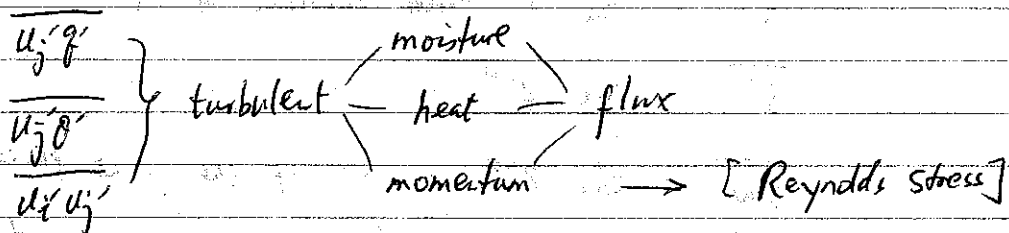
$$\frac{dq}{dt} = K_v \frac{\partial^2 q}{\partial x_j^2}$$

Turbulent Fluxes:

Comparing with $\frac{\partial \bar{q}}{\partial t} + u_j \frac{\partial \bar{q}}{\partial x_j} = K \nabla^2 \bar{q}$

the mean humidity equation $\frac{\partial \bar{q}}{\partial t} + \overline{u_j \frac{\partial q}{\partial x_j}} = K \nabla^2 \bar{q} - \frac{\partial \overline{u_j q'}}{\partial x_j}$

contains a new term $\frac{\partial \overline{u_j q'}}{\partial x_j} \rightarrow$ flux divergences, which act as source terms to change the mean concentration, and which are direct result of the non-linear terms $u_j \frac{\partial q}{\partial x_j}$ in the instantaneous equations.



In ABL, we are only concerned with $\overline{p w' q'}$, $\overline{p w' \theta'}$, $\overline{p u_i' w'}$ [vertical fluxes]

Closure Problem: averaging the equations leads to a situation where there are more unknowns than equations.

Parameterization: representation of the unknown fluxes using resolvable mean variables.

K-closure (1st-order) closure

$$\begin{cases} T_x = -\overline{p u' w'} = \rho K_M \frac{\partial \bar{u}}{\partial z} \\ T_y = -\overline{p v' w'} = \rho K_M \frac{\partial \bar{v}}{\partial z} \\ H_v = \overline{p c_p w' \theta'} = -\rho c_p K_H \frac{\partial \bar{\theta}}{\partial z} \\ E = \overline{p w' q'} = -\rho K_w \frac{\partial \bar{q}}{\partial z} \end{cases}$$

$K_M =$ viscosity
 $K_H =$ eddy thermal diffusivity
 $K_w =$ water vapor

EXERCISE V

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2/11/2002 Deriving governing equations for mean quantity
(Conservation of water vapor)

For the instantaneous specific humidity q , in the absence of phase changes,

$$\frac{dq}{dt} = K_v \frac{\partial^2 q}{\partial x_j^2}$$

$$\text{or } \frac{\partial q}{\partial t} + u_j \frac{\partial q}{\partial x_j} = K_v \frac{\partial^2 q}{\partial x_j^2}$$

Using the Reynolds decomposition,

$$q = \bar{q} + q'$$

$$u_j = \bar{u}_j + u_j'$$

$$\frac{\partial(\bar{q} + q')}{\partial t} + (\bar{u}_j + u_j') \frac{\partial(\bar{q} + q')}{\partial x_j} = K_v \frac{\partial^2(\bar{q} + q')}{\partial x_j^2}$$

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial q'}{\partial t} + \bar{u}_j \frac{\partial \bar{q}}{\partial x_j} + \bar{u}_j \frac{\partial q'}{\partial x_j} + u_j' \frac{\partial \bar{q}}{\partial x_j} + u_j' \frac{\partial q'}{\partial x_j} = K_v \frac{\partial^2 \bar{q}}{\partial x_j^2} + K_v \frac{\partial^2 q'}{\partial x_j^2}$$

Applying "—" on both sides, note

$$\bar{\bar{q}} = \bar{q}, \quad \overline{u_j'} = 0, \quad \overline{q'} = 0, \quad \overline{\frac{\partial q'}{\partial t}} = 0, \quad \overline{\frac{\partial^2 q'}{\partial x_j^2}} = 0$$

$$\overline{\bar{u}_j} = \bar{u}_j, \quad \overline{\frac{\partial q'}{\partial x_j}} = 0,$$

\therefore The ABL flow is assumed incompressible,

$$\frac{\partial u_j}{\partial x_j} = \frac{1}{\rho} \frac{d\rho}{dt} = 0$$

$$\therefore \frac{\partial \bar{u}_j}{\partial x_j} = 0, \quad \frac{\partial u_j'}{\partial x_j} = 0$$

$$\therefore \frac{\partial \bar{q}}{\partial t} + \bar{u}_j \frac{\partial \bar{q}}{\partial x_j} + \frac{\partial \bar{u}_j \bar{q}}{\partial x_j} = K_v \frac{\partial^2 \bar{q}}{\partial x_j^2}$$

$$\therefore u_j' \frac{\partial q'}{\partial x_j} = \frac{\partial (u_j' q')}{\partial x_j} + q' \frac{\partial u_j'}{\partial x_j}$$

(6)

For the horizontally homogeneous ABL, the mean equations

$$\left\{ \begin{aligned} \frac{\partial \bar{u}}{\partial t} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} - \frac{\partial \overline{u'w'}}{\partial z} \\ \frac{\partial \bar{v}}{\partial t} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} - \frac{\partial \overline{v'w'}}{\partial z} \\ \frac{\partial \bar{\theta}_v}{\partial t} &= \frac{1}{\rho c_p} \frac{\partial \bar{R}_v}{\partial t} - \frac{\partial \overline{w'\theta'_v}}{\partial z} \\ \frac{\partial \bar{q}}{\partial t} &= - \frac{\partial \overline{w'q'}}{\partial z} \end{aligned} \right. \implies \frac{\partial \bar{w}}{\partial t} = 0 = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} - g$$

(Hydrostatic equation)

$$f = 2\Omega \sin\phi \quad \text{Coriolis parameter}$$

$$\Omega = 7.29 \times 10^{-5} \text{ rad s}^{-1} \quad \text{the angular velocity of the Earth's rotation}$$

$$\phi = \text{latitude}$$

$$\begin{aligned} f &> 0 \quad \text{N.H.} \\ &< 0 \quad \text{S.H.} \end{aligned}$$

For steady-state conditions, $\frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{v}}{\partial t} = 0$
the momentum equations

$$\left\{ \begin{aligned} 0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} - \frac{\partial \overline{u'w'}}{\partial z} \\ 0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} - \frac{\partial \overline{v'w'}}{\partial z} \end{aligned} \right.$$

Above the ABL, the turbulent terms are zero

$$\left\{ \begin{aligned} 0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} \\ 0 &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} \end{aligned} \right. \implies \text{Geostrophic Balance}$$

pressure gradient terms Coriolis terms

The Geostrophic wind

$$\begin{cases} u_g = -\frac{1}{f\rho} \frac{\partial p}{\partial y} \\ v_g = \frac{1}{f\rho} \frac{\partial p}{\partial x} \end{cases}$$

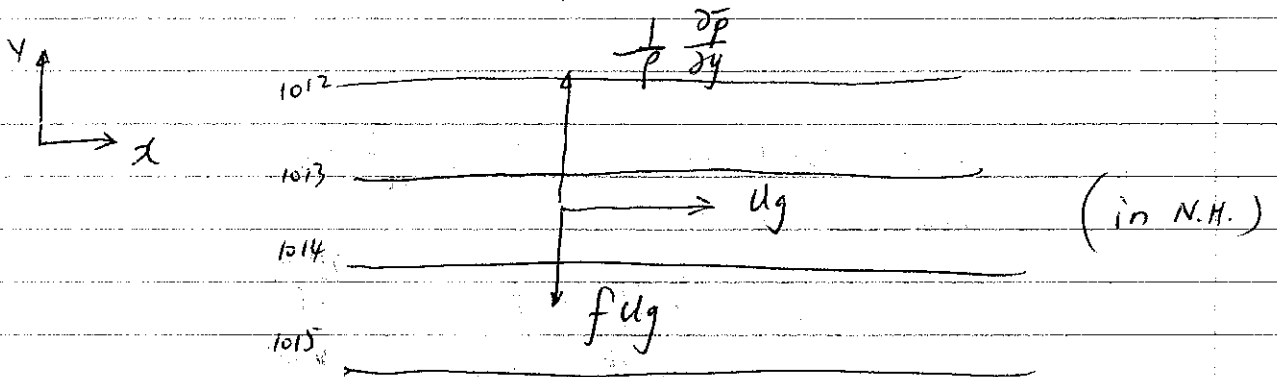
If $|f| = 10^{-4} \text{ s}^{-1}$

$\rho = 1 \text{ kg m}^{-3}$

$\frac{\partial p}{\partial y} = 1 \text{ mb} / (100 \text{ km})$

$$u_g \approx \frac{1}{10^{-4} \times 1} \times \frac{1 \times 10^2 \text{ Pa}}{100 \times 1000 \text{ m}}$$

$$= 10 \text{ m s}^{-1}$$



Ekman Spiral: A useful 1st approximation to the neutral wind profile above the surface layer.
 Consider the momentum equations in the steady-state case,

$$\begin{cases} 0 = f(\bar{v} - v_g) - \frac{\partial \overline{u'w'}}{\partial z} \\ 0 = -f(\bar{u} - u_g) - \frac{\partial \overline{v'w'}}{\partial z} \end{cases}$$

$$-\overline{u'w'} = K_M \frac{\partial \bar{u}}{\partial z}$$

$$-\overline{v'w'} = K_M \frac{\partial \bar{v}}{\partial z}$$

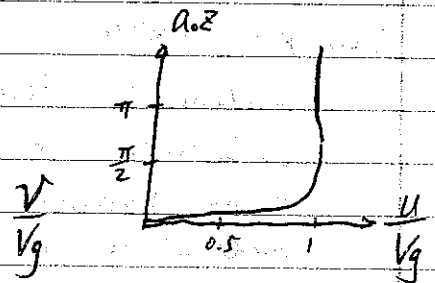
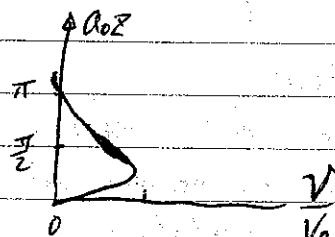
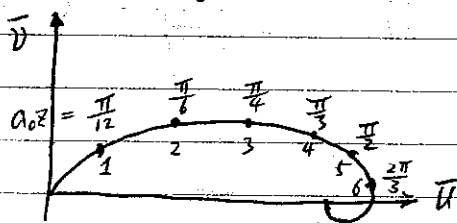
$$\therefore \begin{cases} K_M \frac{\partial^2 \bar{u}}{\partial z^2} = -f(\bar{v} - v_g) \\ K_M \frac{\partial^2 \bar{v}}{\partial z^2} = f(\bar{u} - u_g) \end{cases}$$

Let $\vec{V} = \bar{u} + i\bar{v}$ \rightarrow $\frac{\partial^2 (\bar{u} + i\bar{v})}{\partial z^2} = \frac{f}{K_M} [-(\bar{v} - v_g) + (\bar{u} - u_g)i]$
 $\vec{V}_g = u_g + iv_g$ \rightarrow $\frac{\partial^2 \vec{V}}{\partial z^2} = \frac{if}{K_M} [(\bar{u} - u_g) + (\bar{v} - v_g)i]$

$$\frac{\partial^2 \vec{V}}{\partial z^2} = \frac{if}{K_M} (\vec{V} - \vec{V}_g) \quad \begin{cases} \vec{V} = 0 & z=0 \\ \vec{V} = \vec{V}_g & z=\infty \end{cases}$$

sol. $\begin{cases} \bar{u} = V_g [1 - \exp(-a_0 z) \cos(a_0 z)] \\ \bar{v} = V_g \exp(-a_0 z) \sin(a_0 z) \end{cases}$

$$a_0 = \sqrt{\frac{f}{2K_M}}$$



$$\begin{cases} a_0 z = 0, & \bar{u} = \bar{v} = 0 \\ a_0 z = \frac{\pi}{2}, & \bar{u} = V_g, \bar{v} = V_g \exp(-\frac{\pi}{2}) \\ a_0 z = \pi, & \bar{u} = V_g [1 + \exp(-\pi)], \bar{v} = 0 \end{cases}$$

Roughness length: $\tau_x = -\rho \overline{u'w'}$
 $= \rho u_*^2$
 $= \rho K_M \frac{\partial \bar{u}}{\partial z}$

If $K_M = (Kz)^2 \frac{\partial \bar{u}}{\partial z}$ then $\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{Kz}$

$$\int_0^{\bar{u}} d\bar{u} = \int_{z_0}^z \frac{u_*}{Kz} dz$$

$$\bar{u} = \frac{u_*}{K} \ln \frac{z}{z_0}$$

The logarithmic wind profile \implies generally true for ABL under neutral conditions

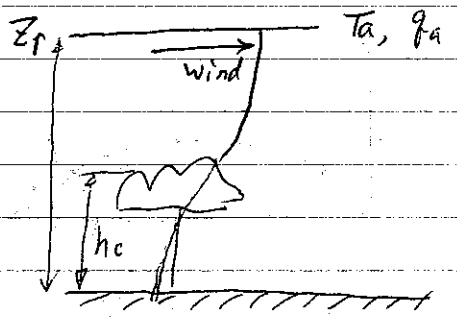
z_0 = the roughness length, a constant of integration chosen so that $\bar{u} = 0$ at $z = z_0$

K = a ~~universal~~ universal constant, von Karman's constant ≈ 0.4

$z_0 = \begin{cases} 1 \text{ mm} & \text{for average seas to more than 1 m} \\ & \text{for cities with tall buildings} \\ 1-4 \text{ cm} & \text{for grassy fields} \\ 0.1 h_c & \text{[between 0.07 and 0.14 } h_c \text{]} \\ & \text{where } h_c = \text{height of canopy} \end{cases}$

Zero-plane displacement d introduced to retain the logarithmic form of the profile in neutral conditions for measurement made above tall vegetation,

$$K \frac{u}{u_*} = \ln \frac{z-d}{z_0}$$



Various forms of latent heat flux

$$LE = L\rho \overline{w'q'}$$

covariance

$$= L\rho C_{pE} U (q_s - q)$$

bulk transfer relations

$$= L\rho \frac{q_s - q}{r_{aE}}$$

aerodynamic resistance relations

} K-theory

$$= L\rho \frac{q^*(T_s) - q_s}{r_s}$$

surface resistance

$$= L\rho \frac{q^*(T_s) - q}{r_s + r_{aE}}$$

$$= \beta_E E_p$$

potential evaporation, $E_p = \rho \frac{q^* - q}{r_{aE}}$

$$= F^*(R_N - G) + (1 - F^*) \frac{\rho}{r^*} \frac{PL \Delta T}{r_{aE}}$$

Penman-Monteith Equation
(combination equation)

Where

$$F^* = \frac{S}{S + \gamma^*} \quad 1 - F^* = \frac{\gamma^*}{S + \gamma^*}$$

$$\gamma^* = \gamma \frac{r_{aE} + r_s}{r_{aE}} = \gamma \left(1 + \frac{r_s}{r_{aE}} \right)$$

$$S = \frac{\partial q^*}{\partial T} \quad \gamma = \frac{c_p}{L}$$

$$q^* = 0.622 \frac{e^*}{p}$$

$$\frac{\partial q^*}{\partial T} = 0.622 \frac{\partial e^*}{\partial T} = q^* \frac{L}{R_v T^2}$$

(See Eqn 4.35)
in GPC

$$r_s = \frac{r_{tot} i}{LAI} \quad LAI = \frac{\sum LAI_i}{A}$$

Surface energy balance $R_n - G_0 = H + LE$

$$R_n = SW^{\downarrow} - SW^{\uparrow} + LW^{\downarrow} - LW^{\uparrow}$$

$$= SW^{\downarrow}(1 - \alpha_s) + \epsilon(LW^{\downarrow} - \sigma T_s^4)$$

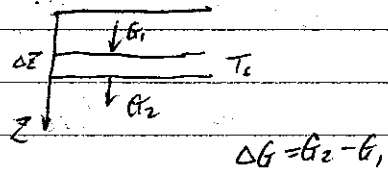
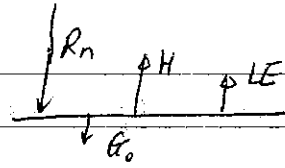
Soil temperature and heat flux

$$\frac{\partial(\rho_s c_s T_s \Delta z)}{\partial t} = -\Delta G$$

$$\Delta z \rightarrow 0, \quad \rho_s c_s \frac{\partial T_s}{\partial t} = -\frac{\partial G}{\partial z}$$

$$G = -K_s \frac{\partial T_s}{\partial z}$$

$$\therefore \frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T}{\partial z^2}$$



specific density \rightarrow heat
 $K_s = \rho_s c_s \alpha_s$
 thermal conductivity thermal diffusivity

Consider diurnal forcing.

$$T_s|_{z=0} = T_0 = \bar{T} + A_0 \sin \Omega t$$

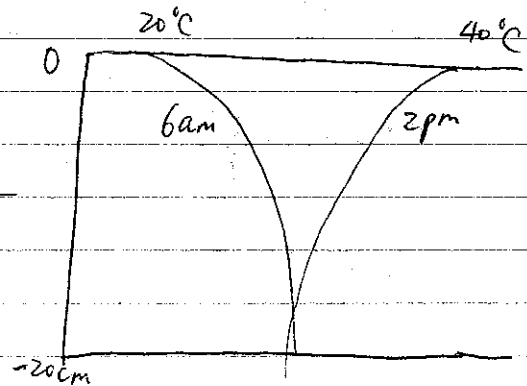
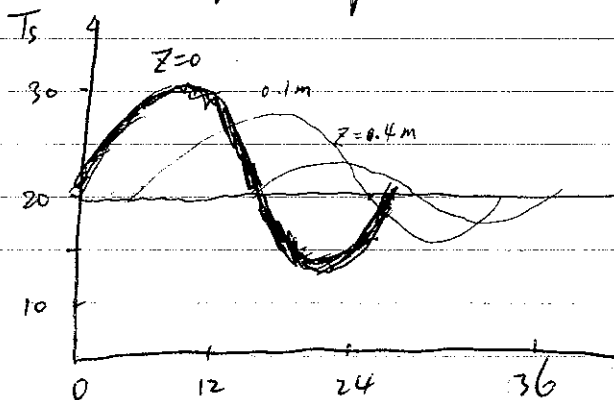
$$T_s|_{z=\infty} = \bar{T}$$

- T_0 = surface temperature
- $2A_0$ = diurnal temp. amplitude
- \bar{T} = daily mean T_s
- $\Omega = 7.292 \times 10^{-5} \text{ rad s}^{-1}$
- $D = \sqrt{\frac{2K_s}{\Omega}}$ damping depth

Solution: $T_s = \bar{T} + A_0 \exp\left(-\frac{z}{D}\right) \sin\left(\Omega t - \frac{z}{D}\right)$

$$G_0 = \rho_s c_s (K_s \Omega)^{\frac{1}{2}} A_0 \sin\left(\Omega t + \frac{\pi}{4}\right)$$

So the soil heat flux is $\frac{\pi}{4}$ out of phase with the temperature wave. Maximum G_0 occurs 3 hours before the maximum surface temperature.



A key variable linking ^{the} surface energy balance (ch. 4) and the water balance (ch. 5) is evaporation.

In ch. 4, the latent heat flux LE , Wm^{-2}

In ch. 5, evaporation (water vapor flux) E , $kgm^{-2}s^{-1}$

Both LE and E are linked by the latent heat of vaporization, L .

$$L = 2.5 \times 10^6 \text{ J kg}^{-1} \quad \text{at } 0^\circ\text{C}$$

$$= 2.25 \times 10^6 \text{ J kg}^{-1} \quad \text{at } 100^\circ\text{C}$$

$$\therefore LE \equiv L \times E$$

SI unit for E is $kgm^{-2}s^{-1}$, but a commonly used unit in hydro-climatology for water fluxes is mm/day .

Below is an example for this conversion:

$$\begin{aligned} \text{If } E &= 1.28854 \times 10^{-5} \text{ kgm}^{-2}\text{s}^{-1} \quad (\text{water vapor flux}) \\ &= \frac{1.28854 \times 10^{-5} \text{ kgm}^{-2}\text{s}^{-1}}{1000 \text{ kgm}^{-3}} \times \frac{1000 \text{ mm}}{1 \text{ m}} \times \frac{86400 \text{ s}}{1 \text{ day}} \\ &= 1.11 \text{ mm/day} \quad [\text{in depth of water per day}] \end{aligned}$$

↓ density of water

This mm/day is also used to describe precipitation, runoff, over a unit area, of course.

$$SH = C_p \rho C_{DH} U (T_s - T_a) ; \quad \frac{\partial SH}{\partial T_s} = C_p \rho C_{DH} U$$

$$B_o \equiv \frac{SH}{LE}$$

{ If the surface is wet, and the air saturated, and assuming $C_{DH} = C_{DE}$, we have

$$B_o = \frac{SH}{LE} = \frac{C_p \cdot (T_s - T_a)}{L \left[Be^{-1} \cdot \frac{C_p}{L} (T_s - T_a) \right]} = Be \quad \}$$

Combination equation (or Penman equation) for E

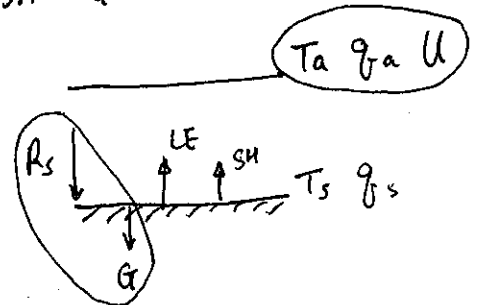
* wet surface \rightarrow the surface air is saturated, $B_o = Be \left[1 - \frac{q_a^* - q_a}{q_s^* - q_a} \right]$

* Bulk aerodynamic formula $E = \rho C_D U (q_s - q_a)$
 $= \rho C_D U (q_s^* - q_a)$

* Surface energy balance $R_s - LE - SH - G = 0$

or $E_{en} = \frac{1}{L} (R_s - G) = E(1 + B_o)$

$$E(1 + Be) = E_{en} + E Be \frac{q_a^* - q_a}{q_s^* - q_a}$$



* Using the bulk formula for E to eliminate E on the right hand side,

$$E = \frac{1}{1 + Be} E_{en} + \frac{Be}{1 + Be} E_{air} \quad \text{where } E_{en} = \frac{R_s - G}{L}$$

$$E_{air} = \rho C_D U (q_a^* - q_a) = \rho C_D U q_a^* (1 - RH)$$

Available energy for turbulent exchanges = $f(R_s, G)$

Atmospheric demand = $f(RH, U, T)$ at one level in the lower atmosphere

Linearization in the surface energy balance
 The blackbody emission from the surface can be linearized
 about air temperature at the reference level,

$$\sigma T_s^4 \approx \sigma T_a^4 + 4\sigma T_a^3 (T_s - T_a) + \dots$$

$$\frac{\partial(\sigma T_s^4)}{\partial T_s} \approx 4\sigma T_a^3$$

Over water or wet land surfaces, the specific humidity at the surface is equal to the saturation specific humidity, q_s^* , at the temperature of the surface

$$q_s = q_s^*(T_s) = q_s^*$$

The specific humidity of saturated air at the reference height can be approximated with a first-order Taylor series.

$$q_a^* = q_s^*(T_a) \approx q_s^*(T_s) + \left. \frac{\partial q_s^*}{\partial T} \right|_{T_s} \cdot (T_a - T_s) + \dots$$

using $RH = \frac{q_a}{q_a^*}$

$$q_a = RH \cdot q_a^* \approx RH \left[q_s^*(T_s) + \left. \frac{\partial q_s^*}{\partial T} \right|_{T_s} (T_a - T_s) \right]$$

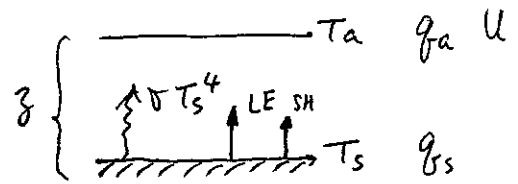
$$\therefore q_s^* - q_a = q_s^* - RH \cdot q_a^* = q_s^* (1 - RH) - RH \left. \frac{\partial q_s^*}{\partial T} \right|_{T_s} (T_a - T_s)$$

$$\therefore LE = L P C_{DE} U (q_s^* - q_a) = L P C_{DE} U (q_s^* - q_a)$$

$$= L P C_{DE} U \left[q_s^* (1 - RH) - RH \left. \frac{\partial q_s^*}{\partial T} \right|_{T_s} (T_a - T_s) \right]$$

$$= L P C_{DE} U \left[q_s^* (1 - RH) + RH Be^{-1} \frac{C_p}{L} (T_s - T_a) \right]$$

where $Be^{-1} = \frac{L}{C_p} \left. \frac{\partial q_s^*}{\partial T} \right|_{T_s} = \frac{L}{C_p} \frac{q_s^*}{T_s} \frac{L}{R_v T_s^2}$



When the surface is not wet, need to consider
the surface resistance.

Penman-Monteith equation.

These ^{combination} equations are useful for measurements or
for calculations when data input is limited.

More elaborate methods as used in LSMs to
account for more physical processes, thereby requiring
more data input. See my review of LSMs
(a powerpoint presentation) and Dr. Fei Chen's overview.
(and a review article)

Analytical Solution for LCL

$$RH = \frac{e}{e^*}$$

$$\ln RH = \ln e - \ln e^*$$

$$d \ln RH = d \ln e - d \ln e^* \quad (1)$$

$$\therefore e = \frac{(\rho P)}{0.622 + 0.378 \rho}$$

$$\therefore \ln e = \ln \frac{\rho}{0.622 + 0.378 \rho} + \ln P$$

For an adiabatic process, $\theta = \text{const}$, $\rho = \text{const}$.

$$d \ln e = d \ln P$$

$$\therefore C_p d \ln \theta = C_p d \ln T - R d \ln P = 0$$

$$\therefore \frac{C_p}{R} d \ln T = d \ln P = \underline{d \ln e} \quad (2)$$

$$\therefore e^* = 6.11 \exp \left\{ \frac{L}{R_v} \left(\frac{1}{273.15} - \frac{1}{T} \right) \right\}$$

$$\therefore \ln e^* = \ln 6.11 + \frac{L}{R_v} \left(\frac{1}{273.15} - \frac{1}{T} \right)$$

$$d \ln e^* = + \frac{L}{R_v T^2} dT \quad (3)$$

$$= \frac{L}{R_v T} d \ln T$$

Substituting (3) and (2) into (1):

$$\int_{RH}^1 d \ln RH = \frac{C_p}{R_d} d \ln T - \frac{L}{R_v T} d \ln T$$

$$= \int_T^{T^*} \left(\frac{C_p}{R_d} - \frac{L}{R_v T} \right) d \ln T$$

$$- \ln RH = \frac{C_p}{R_d} \ln \frac{T^*}{T} + \frac{L}{R_v} \left(\frac{1}{T^*} - \frac{1}{T} \right)$$

Approx:

$$T^* = \frac{1}{\frac{1}{T-55} - \frac{\ln RH}{2840}} + 55$$

T^*, T in K

$$\ln \frac{T^*}{T} = \frac{R_d}{C_p} \ln \frac{P^*}{P}$$

$$P^* = P \left(\frac{T^*}{T} \right)^{\frac{C_p}{R_d}}$$

$T, T^*,$

Nov 1, 2005 Tuesday Quiz #4

[Detail]

Rainfall, averaged over the earth's surface, is about 1 m y^{-1} , or $0.275 \text{ cm day}^{-1}$. On the basis of this information, calculate the following:

(a) The average upward flux of water vapor at the Earth's surface.

Globally & annually,

$$E \equiv P \quad (\text{i.e. evaporation} = \text{precipitation})$$

$$= 0.275 \text{ cm day}^{-1} \quad (\text{the atmospheric storage of water vapor is negligible.})$$

Fig 5.1:

$$P_{\text{total}} = P_{\text{ocean}} + P_{\text{land}}$$

$$= 107 \times 0.7 + 75 \times 0.3$$

$$= 97 \text{ cm}$$

$$E_{\text{total}} = E_{\text{ocean}} + E_{\text{land}}$$

$$= 118 \times 0.7 + 48 \times 0.3$$

$$= 97 \text{ cm}$$

$$= 0.275 \times 10 \text{ mm} \times (86400 \text{ s})^{-1}$$

$$= 3.18 \times 10^{-5} \text{ mm s}^{-1}$$

$$= 3.18 \times 10^{-5} \text{ Kg} \cdot \text{m}^{-2} \text{ s}^{-1}$$

[Note that for water vapor flux, in water equivalent, $1 \text{ mm} \equiv 1 \text{ Kg m}^{-2}$

That is to say, 1 kg of water, if spreaded out over 1 m^2 of area, will have 1 mm thickness.]
 because $1 \text{ kg} / (1000 \text{ kg} \cdot \text{m}^{-3} \times 1 \text{ m}^2) = 10^{-3} \text{ m} = 1 \text{ mm}$ in

(b) The resulting energy flux.

$$\therefore \text{latent heat of vaporization } L = 2.5 \times 10^6 \text{ J kg}^{-1}$$

$$\therefore LE = 2.5 \times 10^6 \text{ J kg}^{-1} \times 3.18 \times 10^{-5} \text{ kg m}^{-2} \text{ s}^{-1}$$

$$= 79.5 \text{ J m}^{-2} \text{ s}^{-1}$$

$$= 79.5 \text{ W m}^{-2}$$

(c) How does this flux compare with the globally averaged incoming solar radiation?

Fig 2.4 compare!!!

The globally averaged incoming solar radiation is $\frac{S_0}{4} = \frac{1367}{4} = 341.8 \text{ W m}^{-2}$

$$\therefore LE / (S_0/4) = 79.5 / 341.8 \approx 23\% \quad \therefore \text{The answer is } 23\% \text{ of it.}$$

(d) If this heat were uniformly distributed within the layer from 900 to 300 mb, what would be the resulting heating rate?

$$\bar{C}_a \cdot \frac{\partial T}{\partial t} = LE \quad \text{where } \bar{C}_a = \frac{\Delta p}{g} \cdot C_p = \frac{(900-300) \times 100}{9.8 \text{ ms}^{-2}} \times 1004 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$= 6146938.8 \text{ J m}^{-2} \text{ K}^{-1}$$

$$\therefore \frac{\partial T}{\partial t} = LE / \bar{C}_a = 79.5 / 6146938.8 \text{ J m}^{-2} \text{ K}^{-1} = 1.293326693 \text{ K s}^{-1} \approx 1.12 \text{ K day}^{-1}$$

$$= 1.12 \text{ }^\circ\text{C day}^{-1}$$

Quiz 4

[sketch]

- (a) Globally and annually, rainfall should be balanced by the water supply to the atmosphere. This is because the atmospheric water storage is so small, with a residence time of only 10 days. Therefore, we can say

$$E \doteq P \doteq 1 \text{ m/year} = 1 \text{ m} \times \frac{1000 \text{ kg}}{1 \text{ m}^3} \times \frac{1}{\text{year}} = 10^3 \text{ kg m}^{-2} \text{ year}^{-1}$$

Proof: Fig 5.1: $P = 107 \times 0.7 + 75 \times 0.3$ [cm]
 $E = 118 \times 0.7 + 48 \times 0.3$ [cm]

- (b) Latent heat flux

$$= E \times \text{latent heat of vaporization}$$

$$= E \times L$$

$$= 10^3 \text{ kg m}^{-2} \text{ year}^{-1} \times 2.5 \times 10^6 \text{ J kg}^{-1}$$

$$= 2.5 \times 10^9 \text{ J m}^{-2} \text{ year}^{-1} = 2.5 \times 10^9 \text{ J} \cdot \text{m}^{-2} (365 \times 24 \times 60 \times 60 \text{ s})$$

$$= 2.5 \times 10^9 / (365 \times 24 \times 60 \times 60) \text{ W m}^{-2}$$

(c) $\frac{LE}{S_0/4} = \frac{2.5 \times 10^9 / (365 \times 24 \times 60 \times 60) \text{ W m}^{-2}}{1367/4 \text{ W m}^{-2}}$

[See Fig 2.4 in Hartmann - p.28
 $LE \sim 24\%$ of incoming solar globally average]

(d) $\Delta m = \frac{\Delta P}{g} = \frac{(900-300) \text{ mb} \times \frac{100 \text{ Pa}}{1 \text{ mb}}}{9.8 \text{ m s}^{-2}}$

$$\bar{C}_a = \Delta m \cdot C_{pa} = \frac{\Delta P}{g} \cdot C_{pa}$$

where $C_{pa} = 1004 \text{ J kg}^{-1} \text{ K}^{-1}$

Energy input = LE , this will cause

heat storage change $\bar{C}_a \frac{\Delta T}{\Delta t}$ 1st Law of Thermodynamics, $LE = \bar{C}_a \frac{\Delta T}{\Delta t}$

$$\therefore \frac{\Delta T}{\Delta t} = LE / \bar{C}_a = \frac{2.5 \times 10^9}{365 \times 24 \times 60 \times 60} \times \frac{1}{\frac{(900-300) \times 100}{9.8} \times 1004}$$

