12.003 Physics of Atmospheres and Oceans
Problem Set 1: Emission temperatures and Greenhouse models – due Friday 21st September, 2007

Read Chapters 1 and 2 of our notes.

1. Suppose that the Earth is, after all, flat. Specifically, consider it to be a thin circular disk (of radius 6370km), orbiting the Sun at the same distance as the Earth; the planetary albedo is 30\%. The vector normal to one face of this disk always points directly towards the Sun, and the disk is made of perfectly conducting material, so both faces of the disk are at the same temperature. Calculate the emission temperature of this disk, and compare with the result we obtained for a spherical Earth.

2. Consider the thermal balance of Jupiter. You will need the following information about Jupiter: mean planetary radius = 69500km; mean radius of orbit around the Sun = 5.19A.U. (where 1A.U. is the mean radius of the Earth’s orbit); planetary albedo = 0.51.

(a) Assuming a balance between incoming and outgoing radiation, calculate the emission temperature for Jupiter.

(b) In fact, Jupiter has an internal heat source resulting from its gravitational collapse. The measured emission temperature $T_e$ defined by

\[ \sigma T_e^4 = (\text{outgoing flux of planetary radiation per unit surface area}) \]

is 130K. Comment in view of your theoretical prediction in part (a). Calculate the magnitude of Jupiter’s internal heat source.

3. For the one-layer “leaky greenhouse” model we considered in class (discussed in Section 2.3.2 of lecture notes), suppose that, all else being fixed, the atmospheric absorption depends linearly on atmospheric $CO_2$ concentration as

\[ \epsilon = \epsilon_0 + [CO_2] \epsilon_1, \]

where $[CO_2]$ is $CO_2$ concentration (in ppm), $\epsilon_0 = 0.734$, and $\epsilon_1 = 1.0 \times 10^{-4}(\text{ppm})^{-1}$. Calculate, for this model, the surface temperature:

(a) for the present atmosphere, with $[CO_2] = 360\text{ppm}$ (see Table 1.2 of notes);
(b) in pre-industrial times, with $[CO_2] = 280\text{ppm}$; and
(c) in a future atmosphere with $[CO_2]$ doubled from its present value.
4. It has been suggested that smoke and fine dust generated by fires resulting from a meteorite hitting the earth could result in intense radiative heating of the mid-troposphere with substantial surface cooling.

(a) Adapt the simple model sketched below (and studied in class) to show that in the extreme case in which the dust layer has zero albedo and is completely absorbing at solar wavelengths, then:

\[
T_s = \left[ \frac{1}{2 - \epsilon} \right]^{\frac{1}{4}} \left[ \frac{S_0}{4\sigma} \right]^{\frac{1}{4}}
\]

where \(\epsilon\) is the absorptivity of the dust layer in the IR.

(b) Insert numbers typical of the Earth — try a range of \(\epsilon\). Discuss briefly [hint: think about the effect of the dust layer on \(\epsilon\)].

5. Consider the “two-slab” greenhouse model illustrated below in which the atmosphere is represented by two perfectly absorbing layers of temperature \(T_a\) and \(T_b\). Determine \(T_a\), \(T_b\), and the surface temperature \(T_s\) in terms of the emission temperature \(T_e\).
12.003 Physics of Atmospheres and Oceans: Sample question and answer

1. For the “simple greenhouse” model we considered in glass we obtained the surface and atmospheric temperatures by considering the energy balance of the surface, and of the Earth-atmosphere system as a whole. Consider the energy budget of the atmosphere alone, and hence show that the solution we obtained is consistent with the atmospheric energy balance.

![Diagram of energy balance](image)

The atmosphere is assumed transparent to solar radiation, so the only input is from absorbed terrestrial radiation

$$\text{net input per unit area} = S \uparrow .$$

The atmospheric layer radiates up and down, so

$$\text{net output per unit area} = A \uparrow + A \downarrow .$$

We found in Chapter 2 of our notes that

$$A \uparrow = A \downarrow = \sigma T_e^4 ,$$
$$S \uparrow = \sigma T_s^4 ,$$
$$T_s = 2^{\frac{1}{2}} T_e .$$

Therefore

$$\text{net input per unit area} = S \uparrow = \sigma T_s^4 = 2\sigma T_e^4 ;$$
$$\text{net output per unit area} = A \uparrow + A \downarrow = 2\sigma T_e^4 ;$$

which are equal. QED.