Approaches and Challenges in Ice Sheet Modelling

Toby Isaac

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The Equations of Motion

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \text{(Continuity)};
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v} \cdot \mathbf{v} \right) = -\nabla \rho + \rho \mathbf{g} + \nabla \cdot \mathbf{T} \quad \text{(Momentum)};
\]

\[
\mathbf{T} = F(\dot{\mathbf{E}}; T, \ldots) \quad \text{(Stress-Strain)};
\]

\[
\dot{\mathbf{E}} = \nabla \mathbf{v} + \nabla \mathbf{v}^T.
\]
The Stokes Approximation

\[ \nabla \cdot \mathbf{v} = 0 \quad \text{(Continuity)}; \]
\[ \nabla p = \rho \mathbf{g} + \nabla \cdot \mathbf{T} \quad \text{(Momentum)}; \]
\[ \mathbf{T} = F(\dot{\mathbf{E}}; T, \ldots) \quad \text{(Stress-Strain)}; \]
\[ \dot{\mathbf{E}} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T. \]

- Good for ice at glacial time scales and pressures
- How is this time dependent?
- Must define \( F \) and boundary conditions
Rheology: defining $F^{-1}$

1. High Stress
   - Viscous, but nonlinear
   - **Glen’s power law**: $\dot{\varepsilon} = A\sigma^3$
   - $A$ increases with increasing temperature

2. Low Stress
   - Viscoelastic
   - *Creep*:
Anisotropy

- Shear easiest in basal plane
- Polycrystalline ice becomes organized with time, pressure, and shear
- Order of magnitude difference in effective viscosity

(Faria, 2003)
Basal boundary conditions

1. Cold boundary
   - No slip: not complicated

2. “Warm” boundary
   - Thin layer of water lubricates
   - **Weertman slip**: \( v = C \sigma^3 \)
   - \( C \) is a parametric roughness constant: how to determine?
   - Is “Stick-Slip” more appropriate?

(Catania, 2008)
Shallow Ice Approximation (SIA)

\[ \partial_x \sigma_{xx} + \partial_y \sigma_{xy} + \partial_z \sigma_{xz} = -\partial_x p; \]
\[ \partial_x \sigma_{xy} + \partial_y \sigma_{yy} + \partial_z \sigma_{yz} = -\partial_y p; \]
\[ \partial_x \sigma_{xz} + \partial_y \sigma_{yz} + \partial_z \sigma_{zz} = -\partial_z p - \rho g; \]
\[ \sigma_{ij} = F(\dot{\varepsilon}_{ij}; T, ...); \]
\[ \dot{\varepsilon}_{ij} = \partial_j v_i + \partial_i v_j. \]
Shallow Ice Approximation (SIA)

\[ \partial_z \sigma_{xz} = -\partial_x p; \]
\[ \partial_z \sigma_{yz} = -\partial_y p; \]
\[ 0 = -\partial_z p - \rho g; \]
\[ \sigma_{ij} = F(\dot{\varepsilon}_{ij}; T,...); \]
\[ \dot{\varepsilon}_{iz} = \partial_z v_i. \]
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\[ \dot{\varepsilon}_{iz} = \partial_z v_i. \]

Hydrostatic approximation \(\Rightarrow\) depth average!
Shallow Ice Approximation (SIA)

\[ \partial_t h = \nabla \cdot [G(\nabla h)] + B \]

\(G\) and \(B\) are dependent on rheology \(F\), boundary conditions, and basal slope.
AR4 draws on four studies to predict changes in ice mass:

1. Huybrechts and de Wolde 1999 (GISM/AISM model + simple atmosphere)
2. Greve 2000 (SICOPOLIS model + simple atmosphere)
3. Huybrechts et al. 2004 (GISM/AISM model forced by GCM)
4. Ridley et al. 2005 (GISM/AISM coupled with GCM)
three, six, and ten degree increase in mean temperature over 1000 years (Greenland only).
Three different CO2 scenarios for next 100 years: wide spread of outcomes
GISM and SICOPOLIS comparison

Remaining ice mass after 1000 years
This difference is due to the much higher ice sheet resolution of GISM and the use of different mass-balance schemes (surface mass balance in GHG1, degree-day in GHG2). As the ice sheet volume declines, surface ablation in GHG2 also declines such that after 2000 yr the freshwater flux in GHG2 has fallen to the same level as in GHG1. One effect of the freshwater input from Greenland is a rise in global average sea level. The direct contribution of the extra water in GHG2 over 3000 yr is around 7 m, with half of this occurring in the first 850 yr. The rate of sea level rise is at a maximum near the start of the experiment, reaching around 5 mm yr$^{-1}$. For comparison, the twentieth-century rate of sea level rise is between 1 and 2 mm yr$^{-1}$, of which less than 0.4 mm yr$^{-1}$ is from Greenland (Church et al. 2001). The rise in Greenland could modify the Atlantic thermohaline circulation (THC) because the freshwater alters high-latitude ocean density, on which the strength of the THC has been shown to depend (Stommel 1961; Rahmstorf 1995; Rind et al. 2001). Such a relation between THC strength and the meridional gradient of salinity between 30°S and 60°N.

A new convection cycle develops, slows melting.
Newer stand-alone models

1. Longitudinal improvements: remove fewer terms
2. Full Stokes where SIA is not appropriate

(Johnson and Staiger, 2007)
Future Directions

▶ Verification (Pattyn, 2008)
▶ Stress history dependence
▶ Rigorous model reduction techniques