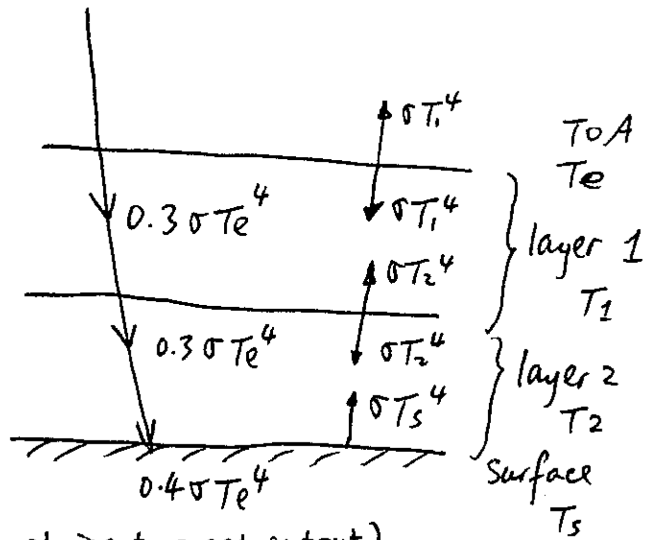
Physical Climatology Problem Set #3

(1) Hartmann Book Page 79 #4:

Figure 3.10 is modified as in the right.



The radiative energy balance (or net input = net output)

at the TOA: $\frac{S_0}{4}(1-\alpha_p) = \sigma T_e^4 = \sigma T_1^4$ ①

in layer 1: $0.3\sigma T_e^4 + \sigma T_2^4 = 2\sigma T_1^4$ ②

in layer 2: $0.3\sigma T_e^4 + \sigma T_s^4 + \sigma T_1^4 = 2\sigma T_2^4$ ③

at the surface: $0.4\sigma T_e^4 + \sigma T_2^4 = \sigma T_s^4$ ④

From ①: $T_1 = T_e = \left[\frac{S_0}{4\sigma}(1-\alpha_p)\right]^{\frac{1}{4}} = 255\ \mathrm{K}$

[or ② + ③ + ④ : $\sigma T_e^4 + \sigma(T_1^4 + 2T_2^4 + T_s^4) = \sigma(2T_1^4 + 2T_2^4 + T_s^4)$

$\sigma T_e^4 = \sigma T_1^4$

$\therefore\ T_1 = T_e$]

From ②: $T_2^4 = 2T_1^4 - 0.3T_e^4 = 2T_e^4 - 0.3T_e^4 = 1.7T_e^4$

$\therefore\ T_2 = (1.7)^{\frac{1}{4}} T_e = 291\ \mathrm{K}$

From ④: $T_s^4 = (0.4 + 1.7)T_e^4 = 2.1T_e^4$

$\therefore\ T_s = (2.1)^{\frac{1}{4}} T_e = 307\ \mathrm{K}$

Following the book in pages 62-63, equation (3.53), the energy balance at the outer edge of the atmosphere,

$$\varepsilon \sigma T_e^4 = 2 \varepsilon \sigma T_{strat}^4$$

$$\therefore T_{strat} = \frac{T_e}{(2)^{\frac{1}{4}}} = 214\ K$$

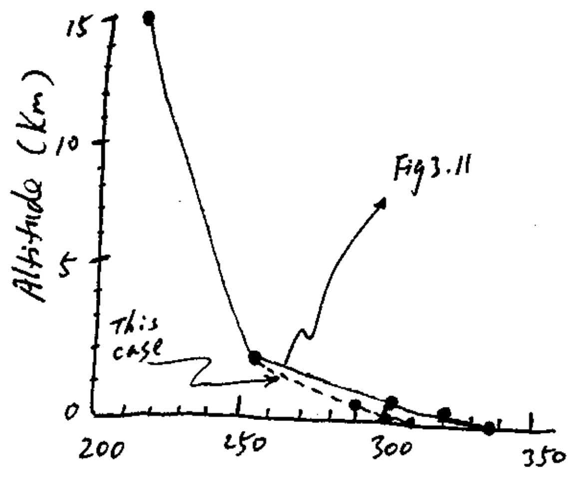
Following equation (3.54), the balance for the air adjacent to the surface,

$$\varepsilon \sigma T_s^4 + \varepsilon \sigma T_2^4 = 2 \varepsilon \sigma T_{SA}^4$$

$$\therefore T_{SA} = \left(\frac{T_s^4 + T_2^4}{2}\right)^{\frac{1}{4}} = \left(\frac{307^4 + 291^4}{2}\right)^{\frac{1}{4}} = 299K$$

Therefore, the radiative equilibrium temperature profile for this case is given as compared to Fig. 3.11

	This case	Fig. 3.11
T_{strat}	214 K	214 K
T_1	255 K	255 K
T_2	291 K	303 K
T_{SA}	299 K	320K
T_s	307K	335K



Conclusion: If all of the solar heating is applied at the surface, the temperatures of the surface and the lower atmosphere are much higher than those when some of the heating is applied. The upper atmosphere, however, is not affected.

(2) Hartmann Book Page 114, #1.

If the top 100 m of ocean warms by 5°C during a 3-month summer period, what is the average rate of net energy flow into the ocean during this period in units of Wm^{-2} ?

$\Delta T = 5°C = 5\ K$

$\Delta t = 3\ \text{month} = 3\ \text{month} \times \frac{30\ \text{day}}{\text{month}} \times \frac{24\ \text{hour}}{\text{day}} \times \frac{60\ \text{min}}{\text{hour}} \times \frac{60\ s}{\text{min}}$

$= 7776000\ s$

$C_{pw} = 4218\ J\ K^{-1}\ Kg^{-1}$ (see p.374)

$\rho_w = 1000\ Kg\ m^{-3}$

The net energy change is

$$100\ m \times \rho_w \times C_{pw} \times \frac{\Delta T}{\Delta t}$$

$$= 100 \times 1000 \times 4218 \times \frac{5}{7776000}$$

$$\doteq 271\ Wm^{-2}$$

If the atmosphere warms by 20°C during the same period, what is the average rate of net energy flow into the atmosphere.

$\Delta T = 20°C = 20K$

$\Delta t = 7776000\ s$

$C_{pa} = 1004\ J\ K^{-1}\ Kg^{-1}$

$P_s = 10^5\ Pa$

$g = 9.81\ ms^{-2}$

The heat capacity of the entire atmosphere (see 4.4)

$$\bar{C_a} = C_{pa} \frac{P_s}{g} = \frac{1004\ JK^{-1}Kg^{-1} \times 10^5\ Pa}{9.8\ ms^{-2}} = 1.02 \times 10^7\ JK^{-1}m^{-2}$$

The net energy exchange is

$$\bar{C_a} \frac{\Delta T}{\Delta t} = 1.02 \times 10^7\ JK^{-1}m^{-2} \times \frac{20\ K}{7776000\ s} \doteq 26\ Wm^{-2}$$